16th French - German - Polish Conference on Optimization

Kraków, Poland, September 23-27, 2013

Book of Abstracts

AGH University of Science and Technology

AGH

Department of Automatics and Biomedical Engineering
Department of Applied Computer Science

16th French-German-Polish Conference on Optimization

Kraków, Poland, September 23-27, 2013

BOOK OF ABSTRACTS

AGH University of Science and Technology

Faculty of Electrical Engineering, Automatics, Computer Science and Biomedical Engineering

Department of Automatics and Biomedical Engineering
Department of Applied Computer Science

Photo by Artur Turyna

Organizing Committee

Ryszard Tadeusiewicz (chair)

Piotr Bania

Adam Korytowski

Janusz Miller

Adam Piłat

Jaromir Przybyło

Anna Sury (secretary)

Maciej Szymkat

Andrzej Turnau

Program Committee

Paul Armand, Limoges, France

Jérôme Bolte, Toulouse, France

Andrzej Cegielski, Zielona Góra, Poland

Hélène Frankowska, Paris, France

Abderrahim Jourani, Dijon, France

Adam Korytowski, Kraków, Poland

Helmut Maurer, Münster, Germany

Stanisław Migórski, Kraków, Poland

Sabine Pickenhain, Cottbus, Germany

Jan Sokołowski, Nancy, France

Maciej Szymkat, Kraków, Poland

Andrzej Świerniak, Gliwice, Poland

Michael Ulbrich, Munich, Germany

Stefan Ulbrich, Darmstadt, Germany

Andrea Walther, Paderborn, Germany

Dariusz Zagrodny, Warsaw, Poland

Previous conferences in this series

15th Austrian-French-German Conference on Optimization, 2011, Toulouse, France
14th Belgian-French-German Conference on Optimization, 2009, Leuven, Belgium
13th Czech-French-German Conference on Optimization, 2007, Heidelberg, Germany
12th French-German-Spanish Conference on Optimization, 2004, Avignon, France
11th French-German-Polish Conference on Optimization, 2002, Cottbus, Germany
10th French-German-Italian Conference on Optimization, 2000, Montpellier, France
9th Belgian-French-German Conference on Optimization, 1998, Namur, Belgium
8th French-German Conference on Optimization, 1996, Trier, Germany
7th French-German Conference on Optimization, 1994, Dijon, France
6th French-German Conference on Optimization, 1991, Lambrecht, Germany
5th French-German Conference on Optimization, 1988, Varetz, France
4th French-German Conference on Optimization, 1986, Irsee, Germany
3rd French-German Conference on Optimization, 1984, Luminy, France
Optimization: Theory and Algorithms, 1981, Confolant, France
Optimization and Optimal Control, 1980, Oberwolfach, W.Germany

Preface

It gives us great pleasure to welcome all participants to the 16th French-German-Polish Conference on Optimization. We hope that it will provide a platform for interchanging ideas, research results and experiences for an international community, actively interested in optimization.

The FGP'13 is the 16th in the series of French-German conferences which started in Oberwolfach in 1980, and since 1998 have been organized with the participation of a third European country. For the second time, it is now organized jointly with Poland. The conference takes place in Kraków, the ancient capital of Polish kings and Poland's oldest university town. While it is full of historical monuments and its Old Town is a well preserved gem of medieval and Renaissance architecture, the city is still a lively cultural center of contemporary Poland.

Kraków is the seat of many scientific institutions and several universities. The oldest of them, the Jagiellonian University had Nicolaus Copernicus among its students. The AGH University of Science and Technology is the biggest technical university in Kraków, with 16 faculties, about 40 000 students and 4 000 staff. Established 100 years ago as an Academy of Mining, it developed into a center of research and education in many advanced areas of modern technology.

The program of the conference includes 10 plenary lectures from various fields of optimization, 3 invited minisymposia and 11 contributed sessions. It is planned that a post-conference edited volume will be published, based on selected conference materials.

This is a good opportunity to thank all persons who helped to organize this conference. We are indebted to the organizers of invited sessions, to the members of the Program Committee and to the reviewers. The success of the conference is largely dependent on the support from the AGH University of Science and Technology, and the Faculty of Electrical Engineering, Automatics, Computer Science and Biomedical Engineering.

We wish you a nice stay in Kraków and fruitful proceedings.

On behalf of the Organizing Committee

Ryszard Tadeusiewicz

Contents

Plenary lectures

Optimal Low Thrust Orbit Transfers with Eclipsing

List of authors:
John T. Betts [1]

Historically there are two steps when placing a satellite in orbit. First, a launch vehicle is used to insert the satellite in a low altitude park orbit, which is followed by an orbit transfer to the final mission orbit. The majority of operational satellites have utilized high thrust chemical propulsion systems to perform the orbit transfer function. A high thrust orbit transfer is characterized by relatively short transfer times, e.g. approximately six hours for a geosynchronous mission. However, high thrust propulsion systems are relatively inefficient, often inserting a payload in the mission orbit that is less than ten percent the weight in the park orbit. The opposite occurs when using modern low thrust propulsion systems, namely greatly improved fuel efficiency at the expense of much longer transfer times. Solar electric propulsion systems deliver these performance benefits, provided the vehicle is located in a region of sunlight so that the engine can operate. When the spacecraft passes through the Earth's shadow (eclipsing), no thrust is generated, and the trajectory design must address this behavior. Furthermore, the location of the shadow changes as a function of time, and since the satellite trajectory may or may not pass through the shadow, the impact of eclipse on the trajectory design can be very significant. Because the thrust acceleration is so small, it is important to include other small perturbations in the dynamic model as well. Finally, the impact of any small force early in the trajectory is greatly amplified because the duration of the transfer may take many months. In short, an optimal low thrust transfer with eclipsing is a a very challenging computational problem.

An approach has been developed for constructing optimal low thrust orbit transfers that addresses the impact of eclipse regions. The complete trajectory is modeled using a sequence of burn and coast phases. An initial guess is constructed by stepping one phase at a time using a receding horizon technique to minimize the orbit error at the end of each phase. The initial guess and phase sequence is then modified during a number of passes that optimize the final mass. The approach is illustrated by computing optimal transfers to geosynchronous and Molniya mission orbits. The overall solution technique requires solving a sequence of over 400 optimal control problems for the example problems.

[1] Applied Mathematical Analysis, LLC

Bregman Iterations in Image Reconstruction

List of authors:

Martin Burger [1]

This talk will discuss variational methods for image reconstruction with nonsmooth regularizations. As an alternative to standard models minimizing the sum of data fidelity and regularization, Bregman iterations have emerged as a state-of-the art yielding superior results in many cases. We shall discuss Bregman iterations for the highly degenerate total variation and sparsity regularizations as well as their analysis. We discuss equivalence or non-equivalence to Augmented Lagrangian formulations and its implications. Finally we present applications in biomedical imaging and an extension to multichannel (color) imaging, which leads to novel questions and further generalizations of Bregman distance concepts.

[1] Working Group Imaging, Institute for Computational and Applied Mathematics, University of Münster, Einsteinstrasse 62, D-48149 Münster, Germany

Optimal control problems for differential inclusions

List of authors:

Piermarco Cannarsa [1]

The typical way to model control systems is the well-known parameterized form $\dot{x} = f(x, u)$ introduced by Pontryagin. A more general representation, however, is the one that describes a control system as a differential inclusion. One may even prefer the latter approach for various questions connected with compactness and existence of optimal solutions. While, by and large, the above approaches seem to be equivalent - thanks to the well-known possibility of parameterizing multifunctions - the existence of smooth parameterizations of a given multifunction is still an open problem. This talk will address typical issues that arise in dynamic programming for nonparameterized control systems, discussing both the Mayer and the minimum time problem. As is natural to expect, our structural assumptions will be given in Hamiltonian form. A surprising difference, with respect to the classical approach to parameterized problems, is the role played by the maximum principle in several regularity issues.

[1] Dipartimento di Matematica Universitá di Roma "Tor Vergata", Via della Ricerca Scientifica 1 00133 Roma, Italy

Attractors for multivalued semiflows governed by subdifferential inclusions

List of authors:

Piotr Kalita [1]

this talk is based on joint work with Grzegorz Łukaszewicz [2]

This talk will address the asymptotic behaviour of the dissipative partial differential inclusions with the multivalued term in the form of the Clarke subdifferential of a locally Lipschitz functional. Such problems can be used as the models of contact phenomena in the mechanics of continuous media. We present three approaches to show the existence of global attractors for formulated problems. For each approach we will show examples of the problems to which this approach applies. In the first approach we will formulate the problem as a *multivalued semiflow (m-semiflow)* and we will show that it is continuous, compact and has a bounded absorbing set. The most difficult of the three properties, the compactness, will be shown by the analysis of the energy function. Moreover, we will define a discrete multivalued semiflow originating from the Rothe method that consists in the time discretization of the continuous problem and we will show that the discrete attractors approximate the continuous one in the sense of upper semicontinuous convergence. The second approach will also use the theory of m-semiflows and it will rely on the relaxation of the compactness assumption to the so called flattening condition and the relaxation of the continuity assumption into the strong weak uppersemicontinuity of the solution map and the weak compactness of its images. We will prove an abstract theorem and provide examples of reaction diffusion problems with multivalued boundary conditions and source terms that follow the presented relaxed framework. The last approach will rely of the theory of trajectory attractors. The global attractor of multivalued semiflow will be obtained as a section of trajectory attractor. We show that the trajectory attractor approach applies to incompressible Navier Stokes flows with multivalued boundary conditions.

[1] Institute of Computer Science, Faculty of Mathematics and Computer Science, Jagiellonian University, ul. prof. S. Lojasiewicza 6, 30-348 Krakow, Poland

[2] Institute of Applied Mathematics and Mechanics, Mathematics Department, University of Warsaw, ul. S. Banacha 2, 02-097 Warsaw, Poland

NONSMOOTH OPTIMIZATION PROBLEMS IN PASSIVE CONTROL OF CRACK PROPAGATION IN LINEAR ELASTICITY

GÜNTER LEUGERING, JAN SOKOŁOWSKI, AND ANTONI ŻOCHOWSKI

1. Shape-topological differentiability of elastic energy in nonsmooth domain

The body with a crack and an inclusion is considered in the framework of linear elasticity. The associated energy functional is used for the control of crack front propagation. The second order mixed shape-topological derivatives of the functional are evaluated for the purposes of passive control of crack front propagation. To this end the Griffith functional is minimized with respect to the admissible family of inclusions.

The unilateral crack model in the framework of linear elasticity is considered in two and three spatial dimensions. The boundary value problem for the elastic displacement field takes the form of a variational inequality over the positive cone in fractional Sobolev space. The variational inequality leads to a problem of metric projection over a polyhedric convex cone, so the conical differentiability concept applies to the shape sensitivity analysis of the variational inequality under considerations.

The specific shape functional associated with the crack form is the so-called Griffith functional. The Griffith functional depends on the singularities of solutions to crack boundary value problems, and it is given by the shape derivative of elastic energy for infinitesimal deformations of the crack front. For our purposes, the Griffith functional is defined by the distributed shape derivative of the energy and it depends exclusively on the solution to the unilateral elasticity boundary value problem in the cracked domain. The sensitivity of the functional is performed for shape-topological perturbations far from the crack by the domain decomposition technique. To this end the conical differentiability of solutions to the variational inequality obtained for the displacement field in the cracked domain is established. The obtained results can be used in the control of crack propagation by the optimum design methods.

In the lecture the domain decomposition technique is employed to singular perturbations in solid mechanics. The singular perturbations of geometrical domains are located far one from another. The domain include a crack Γ_c, so the crack shape is perturbed, the perturbation in the direction of a vector field V is measured by the Griffith shape functional. In order to influence the crack propagation within the elastic body Ω, the small inclusion ω defined by the appropriate caracteristic function χ_ε is introduced far from Γ_c. The influence of the specific inclusion on the crack propagation is measured by the shape-topological derivative of the Griffith functional with respect to the small parameter $\varepsilon \to 0$ which governs the size of the inclusion. The model of the solid body with the crack and inclusion is defined in the nonsmooth domain Ω because of the crack presence, and it is subjected to the singular domain perturbations because of the inclusion ω which reduces to a point in the limit $\varepsilon \to 0$. The displacement field in the elastic body is determined in the framework of linear elasticity from the variational inequality

$$u_\varepsilon \in K(\chi_\varepsilon) \ : \ a(\chi_\varepsilon; u_\varepsilon, v - u_\varepsilon) \geqslant L(\chi_\varepsilon; v - u_\varepsilon) \quad \forall \in K(\chi_\varepsilon) \, . \tag{1}$$

The difficulties associated with the model are

(1) For a fixed parameter $\varepsilon > 0$ the dependence of solutions to the variational inequality on the crack shape perturbations is only Lipschitzian, at most;
(2) The dependence of solutions to variational inequality on singular geometrical domain perturbations can be only modeled in the framework of compound asymptotic expansions;
(3) The second order shape-topological derivative of the energy functional should be evaluated in the direction of two vector fields, the first for the crack perturbations, and the second for the inclusion perturbation;
(4) The second order differentiability of the energy functional is combined with the topological sensitivity analysis in order to determine the sensitivity of Griffith functional with respect to the location of the inclusion.

In conlusion, the direct analysis of the model and derivation of the second order mixed shape-topological derivative could be out of the scope of modern mathematical theory. That is why, we are going to obtain the new, positive results by employing the domain decomposition technique to our model. Therefore, the domain Ω with the crack and the inclusion is decomposed into $\Omega := \Omega_c \cup \Gamma \cup \Omega_R$ where

- Ω_c contains the crack, so we are going to impose the unilateral conditions on the crack in the subproblem defined in Ω_c;
- Ω_R contains the inclusion, so we are going to perform the asymptotic analysis associated with the small parameter ε in the subproblem defined in Ω_R;
- two subproblems are coupled via the interface Γ with the appropriate Steklov-Poincaré operators.

In the paper *in preparation* the first and the second order nonsmooth shape sensitivity analysis is performed for the elastic energy functional in domains with cracks. Such an analysis can be found in the monograph [7] for the frictionless contact problem of an elastic body with the rigid foundation, the so-called the Signorini problem in linear elasticity.

The crack is considered in the framework of linear elasticity, however the nonlinear boundary conditions of unilateral type are prescribed on the crack itself. This makes the solution map for crack boundary value problem nonsmooth, since the solution of the boundary value problem is only Lipschitz continuous with respect to the data, e.g., with respect to the right-hand side of the variational inequality given by the loading applied to the elastic

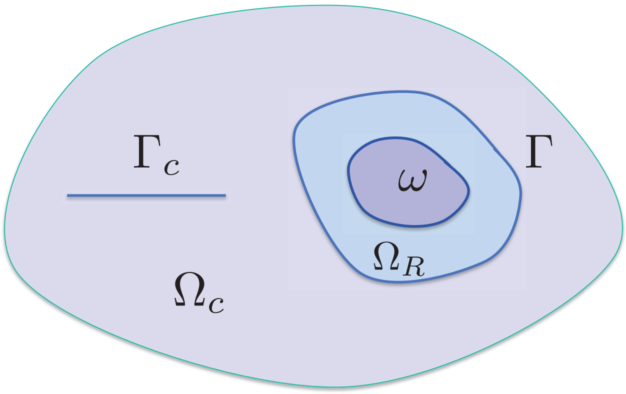


FIGURE 1. Domain Ω with crack and inclusion

body under considerations. However, the elastic energy functional is differentiable with respect to the boundary variations including the perturbations of the crack tip. We are interested in a specific problem of the second order shape-topological differentiability of the elastic energy functional

- the first order differentiability, with respect to crack shape,
- then the second order differentiability of the resulting shape functional with respect to boundary-topological variations far from the crack.

As a result it is shown that the *Griffith shape functional* is differentiable with respect to the domain variations far from the crack, and the derivative is explicitely determined.

REFERENCES

[1] FRÉMIOT, GILLES; HORN, WERNER; LAURAIN, ANTOINE; RAO, MURALI; SOKOLOWSKI, JAN On the analysis of boundary value problems in nonsmooth domains. Dissertationes Math. (Rozprawy Mat.) 462 (2009), 149 pp.

[2] G. LEUGERING, J. SOKOŁOWSKI, A. ŻOCHOWSKI *Shape-topological differentiability of energy functionals for unilateral problems in domains with cracks and applications*, to appear.

[3] A.M. KHLUDNEV, J. SOKOŁOWSKI, *Modelling and control in solid mechanics,* Basel-Boston-Berlin, Birkhäuser, 1997, reprinted by Springer in 2012.

[4] A.A. NOVOTNY, J. SOKOŁOWSKI, *Topological Derivatives in Shape Optimization*, Series: Interaction of Mechanics and Mathematics, Springer-Verlag, 2013, 433 p.

[5] PLOTNIKOV, PAVEL, SOKOLOWSKI, JAN Compressible Navier-Stokes equations. Theory and shape optimization. Instytut Matematyczny Polskiej Akademii Nauk. Monografie Matematyczne (New Series) [Mathematics Institute of the Polish Academy of Sciences. Mathematical Monographs (New Series)], 73. Birkhäuser/Springer Basel AG, Basel, 2012. xvi+457 pp.

[6] SOKOLOWSKI, J., ŻOCHOWSKI, A. Modelling of topological derivatives for contact problems. Numer. Math. 102 (2005), no. 1, 145âĂŞ179.

[7] SOKOŁOWSKI, J., ZOLÉSIO, J.-P., *Introduction to Shape Optimization. Shape Sensitivity Analysis*, Springer Ser. Comput. Math. 16, Springer, Berlin, 1992, reprinted in 2013.

Department Mathematik, Friedrich-Alexander-Universität Erlangen-Nürnberg, Cauerstr. 11 (03.322) 91058 Erlangen, Germany
E-mail address: `leugering@math.fau.de`

Institut Élie Cartan, UMR 7502 Nancy-Université-CNRS-INRIA, Laboratoire de Mathématiques, Université Henri Poincaré Nancy 1, B.P. 239, 54506 Vandoeuvre Lès Nancy Cedex, France, and Systems Research Institute of the Polish Academy of Sciences, ul. Newelska 6, 01-447 Warszawa, Poland
E-mail address: `Jan.Sokolowski@iecn.u-nancy.fr`

Systems Research Institute of the Polish Academy of Sciences, ul. Newelska 6, 01-447 Warszawa, Poland
E-mail address: `Antoni.Zochowski@ibspan.waw.pl`

Optimal control for medical applications - challenges and solutions

List of authors:
K. Mombaur [1]

Optimal control problems are ubiquitous in medical applications, in particular in the field of motion generation. This talk serves to highlight challenges for applied mathematics, and especially optimization and optimal control arising from this interesting area of applications and to present potential solution approaches. The goals in this context are to better understand human movement and to use this understanding to improve the movement either by controlling it directly, by developing better training and rehabilitation techniques, or by optimally designing technical devices that support or guide the movement.

We will discuss different examples for optimal control problems in medical applications, e.g.

- the generation of optimal muscle stimulation patterns for functional electrical stimulation in walking, in particular for the treatment of the drop foot syndrome of stroke patients
- the analysis and improvement of stability of normal and pathological gait
- the optimization-based development and control of an exoskeleton for medical applications
- the study of walking and running motions with prostheses, e.g. in the context of disability sports: high-speed running of bipedal amputees on carbon fiber prostheses
- optimal sit to stand transfer with and without physical assistive devices.

Among the mathematical challenges arising from these applications are:

- The efficient and flexible modeling of these complex biomechanical systems, sometimes in combinations with the respective technical devices. The mathematical descriptions of such motions result in highly nonlinear systems of ordinary differential or differential-algebraic equations, generally including multiple phases of motion, implicitly defined phase changes and discontinuities of state variables between phases. These models need to be adjustable to different subjects and situations, and the right level of complexity has to be chosen for each application. The identification of good data for human models also present a big issue.
- A correct formulation of optimal control problems for the generation and control of such motions: This generally results in a hybrid multi-phase optimal control problem including switches, continuous and discrete phases, constraints and objective functions. Avoiding global and local redundancy of the constraints poses a particular challenge in some applications. Objective functions can get very complex as soon as stability issues are involved:

[1] ORB, Interdisciplinary Center for Scientific Computing, University of Heidelberg, Germany

in this case derivatives of the trajectories have to be considered in the objective function or constraint formulations, and the variational differential equation of the hybrid dynamics have to be included in the dynamic constraints of the problem.

- Initialization of state and control variables: due to the large number of variables an automated initialization is favorable, and due to the local nature of of the optimization procedures, a good initialization is very important.

- An efficient solution of optimal control problems is essential, both for offline (generation/ selection of motions) and online problems (control of motions). Direct optimal control techniques using multiple shooting have proven very efficient for the solution of such problems. While in the offline case, precise solutions for whole body human models can be sought for, reduced models and real-time methods have to be used in the online case.

- An efficient solution of inverse optimal control problems: Inverse optimal control problems are formulated to identify optimization objectives of motions from (partial) measurements of state variables and potentially control variables. This class of problems is particularly challenging, since it consists in solving a parameter estimation in an optimal control problem. Bi-level as well as one-level methods have been developed to solve this type of problems.

- Handling of uncertainties and variability in data: data in this context is recorded by optical motion tracking systems, inertial measurement units, force plates, EMG etc. None of these measurements is precise. In addition, there is a lot of variation between subjects, motion trials, scenarios, ... Deciding which data can be combined for which analysis (e.g. which motions are combined in one inverse optimal control computation with the hypothesis that they share the same underlying objective function) is a very hard problem.

- The transfer of optimization results to reality also is an issue. Once optimal motions have been computed for a prostheses, and exoskeleton, another physical assistive device, or a stimulation pattern, they have to be applied to the real system, and methods for coping with the model mismatch are required.

No-Gap Necessary and Sufficient Second Order Conditions in Optimal Control: Theory and Applications

List of authors:

N. Osmolovskii [1]

H. Maurer [2]

We observe some results contained in our monograph published by SIAM in 2012 [2]. First we formulate no-gap necessary and sufficient second order conditions in optimal control problems with ordinary differential equations considered on a non-fixed time interval, subject to endpoints and mixed state-control constraints. The conditions admit discontinuities of the first kind of the reference control and take them into account. Next we formulate such conditions for optimal control problems with a vector control variable having two components: a continuous unconstrained control appearing nonlinearly in the control system and a bang-bang control appearing linearly and belonging to a convex polyhedron. Such type of control problem arises in many applications. Particular emphasis is given to bang-bang control problems. Bang-bang controls induce an optimization problem with respect to the switching times of the control. It turned out that the classical second-order sufficient condition for the Induced Optimization Problem (IOP), together with the so-called strict bang-bang property, ensures second-order sufficient conditions (SSC) for the bang-bang control problem. We mention a number of numerical examples in different areas of application which illustrate the verification of SSC for both regular controls and bang-bang controls.

References

[1] Milyutin, A.A., Osmolovskii, N. P., Calculus of variations and optimal control. Translations of mathematical monographs, vol. **180**, American Mathematical Society, 1998.

[2] Osmolovskii, N.P. and Maurer, H., Second-Order Necessary and Sufficient Optimality Conditions in Calculus of Variations and Optimal Bang-Bang Control: Applications to Regular and Bang-Bang Control. Society for Industrial and Applied Mathematics, Philadelphia, USA, 2012.

[1] the University of Technology and Humanities in Radom, Poland

[2] Westfälische Wilhelms-Universität, Münster, Germany

New Optimality Conditions and Methods for State-Constrained Elliptic Optimal Control Problems

List of authors:

Hans Josef Pesch [1]

joint work with M. Frey, S. Bechmann, and A. Rund

Based on two different reformulations of the state constraints and a hypothesis on the structure of the active set, new necessary conditions for linear-quadratic elliptic optimal control problems with distributed controls are obtained which exhibit higher regularity of the multipliers associated with the state constraint. Moreover, we obtain also new jump and sign conditions. Measures are no longer an issue. The two investigated approaches mimic the well-known Bryson-Denham-Dreyfus indirect adjoining method which is the preferred ansatz in solving state constrained optimal control problems with ordinary differential equations numerically. Mathematically the reformulations lead to a new kind of set optimal control problem, where the active set of the state constraint, resp. the interface between the inactive and the active set are to be determined as part of the solution. Various formulations of this type of optimization problem as bilevel optimization problems are discussed which also include shape-optimization. Moreover, parallels can be drawn to optimization on vector bundels. In the end, these conditions can be formulated as free boundary value problems. Numerical results will demonstrate the performance of the resulting numerical method.

[1] University of Bayreuth, Chair of Mathematics in Engineering Sciences

A distance for probability spaces, and long-term values in MDP and Repeated Games

List of authors:
Jérôme Renault [1]

joint work with Xavier Venel (Tel-Aviv University)

Given a compact subset X of a normed vector space, we study the pseudo-metric on Borel probabilities over X given by $d_*(u,v) = \sup_{f \in D_1} |u(f) - v(f)|$, where D_1 is the set of functions satisfying: $\forall x, y \in X, \forall a, b \geq 0,\ af(x) - bf(y) \leq \|ax - by\|$. The particular case where X is a simplex endowed with the L^1-norm is particularily interesting: in this case d_* can be characterized as the largest distance on the probabilities with finite support over X which makes all disintegrations non expansive. Moreover we obtain a Kantorovich-Rubinstein type duality formula for $d_*(u,v)$ involving couples of measures (α, β) over $X \times X$ such that the first marginal of α is u and the second marginal of β is v.

In the second part of the paper, we study several kinds of Markov Decision Processes, Gambling Houses and 2-player zero-sum Repeated Games. It includes in particular all partial observation MDPs with finitely many states, and repeated games with an informed controller and finite sets of states and actions. In each case, the underlying state space is compact and the transitions can be shown to be non expansive for the distance d_* (or in some case for the Kantorovich-Rubinstein distance). This allows us to prove the existence of, and to characterize via the introduction of appropriate invariant measures, a very strong notion of limit value called the general uniform value. The decision-maker is able to play well independently of the time horizon, and regarding evaluations for payoffs we consider not only the Cesaro means when the number of stages is large, but any evaluation function θ over stages when the total variation, or impatience, $TV(\theta) = \sum_{t \geq 1} |\theta_{t+1} - \theta_t|$ is small enough.

[1] MF 414, GREMAQ, TSE et Département de Maths, Manufacture des Tabacs, Université Toulouse 1 Capitole, 21 allée de Brienne, 31000 Toulouse, France

Evaluation complexity in nonlinear optimization

List of authors:

Philippe Toint [1]

(with C. Cartis and N. Gould)

The talk will attempt to cover a number of recent results in the worst-case complexity analysis of algorithms for smooth nonlinear (and potentially non convex) optimization problems, both constrained and unconstrained. The first part will investigate the evaluation complexity of the standard problem and show some surprising results concerning classical methods, as well as the remarkable properties of the ARC methods, which are based on a cubic regularization scheme. It will also be shown that some of these results are sharp or optimal by discussing examples where the worst-case behaviour of the considered methods is actually achieved. Building upon the analysis for the unconstrained case, it will also be shown how to extend the analysis to the case of convexly constrained problems first, and finally to the general problem involving nonlinear equalites and inequalities. A discussion of some conclusions and remaining challenges will be proposed.

[1]Department of Mathematics, The University of Namur (FUNDP), 61, rue de Bruxelles, B5000 - Namur, Belgium

Minisymposium on Image Processing and Inverse Problems

organized by Marcus Wagner and Christoph Brune

Shape from Shading: Classic and Modern Methods in Computer Vision

List of authors:
M. Breuß [1]
Y.C. Ju [2]

Shape from shading (SFS) is a classic inverse problem from computer vision with many potential real-world applications. Given a single input image, the task of SFS is to determine the shape of a depicted object at hand of assumptions on lighting and light reflectance in the scene. For an overview see [3].

Based on the underlying assumptions, the SFS methods differ a lot with respect to the mathematical properties of the models. In recent years, the so-called perspective SFS models became relatively popular, see e.g. [4]. These are useful in settings where the camera is relatively close to photographed objects. Making use of specific combinations of modelling assumptions, one can prove well-posedness properties of the respective models [1]. Concerning mathematical formulations that arise with SFS, Hamilton-Jacobi partial differential equations (PDEs) as well as related optimal control approaches are often employed [2]. However, in classic works as well as for recent modeling extensions the flexibility of variational methods is appreciated.

After an introduction to the issues arising in SFS, we will consider some classic methods in the field as well as recent optimization approaches. The goal of our presentation is to show that computer vision is an active and interesting field of research that offers interesting applications for applied mathematics.

References

[1] M. Breuß, E. Cristiani, J.-D. Durou, M. Falcone and O. Vogel Perspective Shape from Shading: Ambiguity Analysis and Numerical Approximations SIAM J. Imaging Sciences 5(1):311–342, 2012

[2] M. Breuß, E. Cristiani, J.-D. Durou, M. Falcone and O. Vogel Numerical Algorithms for Perspective Shape from Shading Kybernetika 46:207–225, 2010

[3] J.-D. Durou, M. Falcone and M. Sagona Numerical methods for shape-from-shading: A new survey with benchmarks Computer Vision and Image Understanding 109(1):22–43, 2008

[4] O. Vogel, M. Breuß and J. Weickert Perspective shape from shading with non-Lambertian reflectance In G. Rigoll (Ed.): Pattern Recognition. Lecture Notes in Computer Science 5096:517–526, Springer, Berlin, 2008.

[1] Applied Mathematics and Computer Vision Group, Brandenburg University of Technology Cottbus, Germany
[2] Applied Mathematics and Computer Vision Group, Brandenburg University of Technology Cottbus, Germany

Image Reconstruction and Wasserstein Transport Models for 4D Blood Cell Transmigration

List of authors:

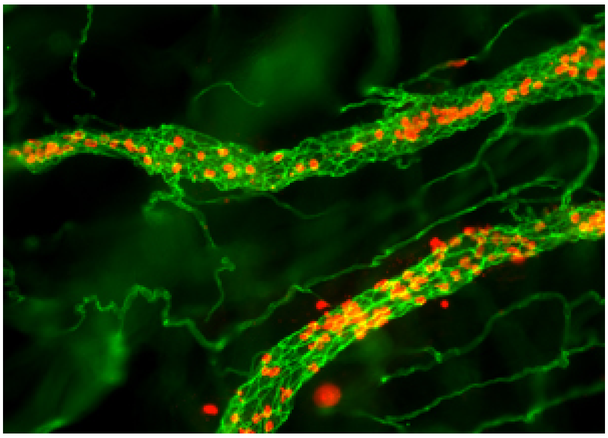
Christoph Brune [1]

Martin Burger [2]

Dietmar Vestweber [3]

Understanding and controlling leukocyte (white blood cell) extravasation is a very important goal for cell biology and medical treatment during inflammation (see figure below). It is related to autoimmune or hardly curable chronic diseases e.g. arthritis, periodontitis or multiple sclerosis. A fundamental open question is the specific pathway leukocytes take (trans- or paracellular migration) and their mechanical constraints leaving and entering blood vessels.

In this talk we will present recent developments for combined motion estimation and reconstruction models for tackling time-dependent inverse problems in 4D bioimaging. In particular, we will focus on modeling variational methods under parabolic PDEs including conservations laws and sparsity constraints. We will address optimal Wasserstein transport for combined tracking, denoising and deblurring of cells in motion and present achievements for the analysis and optimization of those models. Synthetic and experimental numerical results will underline the importance of understanding and simulating cell transmigration through barriers.



[1]Institute for Computational and Applied Mathematics, University of Münster, Germany
[2]Institute for Computational and Applied Mathematics, University of Münster, Germany
[3]Max-Planck-Institute for Molecular Biomedicine, Münster, Germany

TGV Based Image Reconstruction: Analytics

List of authors:

M. Holler [1]

K. Bredies [2]

In JPEG compression for digital images, data reduction is achieved by quantizing the coefficients of a cosine transform of the original image. As a result, the source image cannot be obtained uniquely from the compressed JPEG file and the decompressed image typically suffers from compression artifacts. Motivated by the aim of reducing such artifacts, an optimization problem for multichannel image reconstruction from inexact or incomplete data is considered. The task is to minimize the sum of two convex functionals, one ensuring data fidelity and the other one being a regularization term. Having the application to transform based image coding such as JPEG in mind [H12a, H12b], point-wise interval restrictions on the Riesz-basis transformed images are imposed for data fidelity. For regularization, the Total Generalized Variation (TGV) [B10] functional of arbitrary order is used.

A formulation of the optimization problem in a general function space setting is presented. This also allows further applications such as improved JPEG 2000 decompression and wavelet based zooming. The results of [B11] are generalized to the TGV functional of arbitrary order and used to obtain existence of a solution and optimality conditions. At last, based on the primal-dual algorithm of [C11], numerical results for the desired applications are provided, in particular showing the ability of the considered approach to obtain high quality reconstructions even from strongly compressed JPEG files.

References

[B10] K. Bredies, K. Kunisch and T. Pock *Total generalized variation.* SIAM J. Imag. Sci., 2010.

[B11] K. Bredies and T. Valkonen *Inverse problems with second-order total generalized variation constraints.* Proceedings of SampTA, 2011.

[C11] A. Chambolle and T. Pock *A first-order primal-dual algorithm for convex problems with applications to imaging.* J. Math. Imaging Vision, 2011.

[H12a] K. Bredies and M. Holler *Artifact-free decompression and zooming of JPEG compressed images with total generalized variation.* Springer, Commun. Comput. Inf. Sci., 2012.

[H12b] K. Bredies and M. Holler *A Total-Variation-based JPEG decompression model.* SIAM J. Imag. Sci., 2012.

[1] University of Graz

[2] University of Graz

A Convex Variational Approach for Restoring Data Corrupted with Poisson-Gaussian Noise

List of authors:

A. Jezierska [1]

E. Chouzenoux[1]

J.-C. Pesquet[1]

H. Talbot[1]

The Poisson-Gaussian (PG) model is well suited to a number of imaging systems. The Poisson component is often related to the quantum nature of light and accounts for photon-counting principles in signal registration, whereas the Gaussian component is typically related to thermal noise present in the electronic part of the imaging system. Despite constant improvements in data acquisition devices, electronic noise cannot usually be neglected. However, up to now, PG model had not been widely used because of theoretical and practical difficulties. Among existing works dealing with PG noise, a number of methods have addressed image restoration problems, but they rely on approximations of the noise statistics. In view of this, we formulate the image restoration problem in the presence of PG noise in a variational framework, where we express and study the exact data fidelity term. After establishing the Lipschitz differentiability and convexity of the exact PG neg-log likelihood, we derive a primal-dual optimization algorithm for the reconstruction of images degraded by a linear operator and corrupted with PG noise. Using recent primal-dual convex optimization algorithms, we obtain results that outperform methods relying on a variety of approximations. The proposed approach is validated on image restoration examples.

References

[1] A. Jezierska, E. Chouzenoux, J.-C. Pesquet, and H. Talbot, A primal-dual proximal splitting approach for restoring data corrupted with Poisson-Gaussian noise

International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Kyoto, Mar. 2012.

[2] P. L. Combettes, and J.-C. Pesquet, Primal-Dual Splitting Algorithm for Solving Inclusions with Mixtures of Composite, Lipschitzian, and Parallel-Sum Type Monotone Operators, *Set-Valued and Variational Analysis*, vol 20., pp 307–330, 2012.

[1] Université Paris-Est, Laboratoire d'Informatique Gaspard Monge, CNRS-UMR 8049, 77454 Marne-la-Vallée Cedex 2, France. Phone: +33 1 60 95 77 39, E-mail: {first.last}@univ-paris-est.fr.

Image denoising with Kantorovich-Rubinstein discrepancy

List of authors:
D. Lorenz [1]

One of the most prominent models for image denoising is the Rudin-Osher-Fatemi model [3], in which one minimizes, for a given $u^0 \in L^2(\Omega)$ the functional

$$\int_\Omega (u - u^0)^2 \, dx + \lambda TV(u)$$

with some $\lambda > 0$ and TV denoting the total variation of u. Although the method does remove noise and preserves edges well, some drawbacks of this model has been identified, namely a loss of contrast and the staircasing effect. The loss of contrast can be overcome by either using a so-called Bregman-iteration or by changing the discrepancy term to a one norm as proposed by Chan and Esedoglu [2]: For a given $u^0 \in L^(\Omega)$ one minimizes

$$\int_\Omega |u - u^0| \, dx + \lambda TV(u).$$

This method indeed preserves the contrast als also enjoys a kind of contrast invariance. However, staircasing still occurs.

With respect to staircasing, several adaptions of the TV seminorm has been proposed, most notably the total generalized variation TGV by Bredies, Kunisch and Pock [1].

In this talk we will leave the domain of function spaces and model images in the space $\mathfrak{M}(\Omega)$ of Radon measures. The space $L^1(\Omega)$ is embedded in $\mathfrak{M}(\Omega)$ and it even holds that $\|f\|_1 = \|f\|_{\mathfrak{M}}$. However, there are other norms on the space of Radon measures which capture more geometric information. One example is the Kantorovich-Rubinstein norm which is defined by duality as

$$\|\mu\|_{KR} = \sup\{\int f \ d\mu \ : \ |f| \leq 1, \ \mathrm{Lip}(f) \leq 1\}$$

(where $\mathrm{Lip}(f)$ stands for the Lipschitz constant of f). The Kantorovich-Rubinstein norm induces a metric $d_{KR}(\mu, \nu) = \|\mu - \nu\|_{KR}$ which does metrize weak-* convergence of Radon measure which is strictly weaker than norm convergence.

In this talk we will show how the Kantorovich-Rubinstein norm can be used as dicrepancy term for total variation denoising, i.e. we will consider the minimization of

$$\|u - u^0\|_{KR} + \lambda TV(u).$$

We present a primal dual method to solve the minimization problem and it turns out that the resulting algorithm is only slightly more involved than for the Chenc-Esedoglu model. Numerical examples indicate that the proposed model does not suffer from contrast lost and does produce much less staircasing that the Rudin-Osher-Fatemi and the Chan-Esedoglu model.

[1] Institute for Analysis and Algebra, TU Braunschweig, 38092 Braunschweig, Germany

References

[1] Kristian Bredies, Karl Kunisch and Thomas Pock Total generalized variation. *SIAM Journal on Imaging Sciences*, 3(3):492-526, 2010.

[2] Tony F. Chan, Selim Esedoglu Aspects of total variation regularized L^1 function approximation. *SIAM J. Appl. Math.*, 65:1817-1837, 2005.

[3] Leonid I. Rudin, Stanley J. Osher, and Emad Fatemi. Nonlinear total variation based noise removal algorithms. *Physica D*, 60:259–268, 1992.

An entropy model for diffusion MRI

List of authors:

Pierre Maréchal [1]

Diffusion MRI has developped in the recent past as a method for non-invasive imaging of the diffusion process of water in biological tissues. Since diffusion is affected by obstacles such as membranes and fibers, diffusion MRI opens the way to mapping fiber structures (e.g. neurons or muscle fibers). The diffusion MRI data are samples of the Fourier transform of the probability density of particle displacements in each voxel of the volume to be imaged. The sampling is generally poor: the probability density must be inferred from a few dozens of Fourier samples. The idea of using the maximum entropy principle in this context appeared, to the best of our knowledge, in a paper published in 2005 by D. Alexander [1]. In this paper, the displacement is assumed to be confined to a sphere centered at the intitial point of the particle. In order to better suit the physics of the diffusion process, we propose to relax this constraint and build a more general methodology. The Kullback-Leibler relative entropy is used to measure the discrepancy between the probability to be inferred and some reference measure. The obtained optimization problem is then studied usind tools from partially infinite convex progamming. The solution can be computed via the unconstrained maximization of a smooth concave function, whose number of variable is merely (twice) the number of Fourier samples.

References

[1] D.C. Alexander, *Maximum entropy spherical deconvolution for diffusion MRI*, Proc. Information processing in medical imaging, 2005.

[1] Université de Toulouse, ISAE

Optical flow with oscillating pattern

List of authors:
A. R. Patrone [1]
O. Scherzer [2]

In this paper we present a variational method for determining cartoon and texture components of the optical flow[2] of a noisy image sequence. The method is realized by applying decomposition methods[1] and then by using spatio-temporal regularizers[4][5]. We study a decomposition for the optical flow into bounded variation and oscillating component[3] in greater detail. Numerical examples demonstrate the capabilities of the proposed approach.

References

[1] J. Abhau, Z. Belhachmi, and O. Scherzer. On a decomposition model for optical flow. In *Energy Minimization Methods in Computer Vision and Pattern Recognition*, volume 5681 of *Lecture Notes in Computer Science*, pages 126–139. Springer-Verlag, Berlin, Heidelberg, 2009.

[2] B. K. P. Horn and B. G. Schunck. Determining optical flow. *Artificial Intelligence*, 17:185–203, 1981.

[3] Y. Meyer. *Oscillating patterns in image processing and nonlinear evolution equations*, volume 22 of *University Lecture Series*. American Mathematical Society, Providence, RI, 2001. The fifteenth Dean Jacqueline B. Lewis memorial lectures.

[4] J. Weickert and C. Schnörr. A theoretical framework for convex regularizers in PDE-based computation of image motion. *Int. J. Comput. Vision*, 45(3):245–264, 2001.

[5] J. Weickert and Ch. Schnörr. Variational optic flow computation with a spatio-temporal smoothness constraint. *J. Math. Imaging Vision*, 14:245–255, 2001.

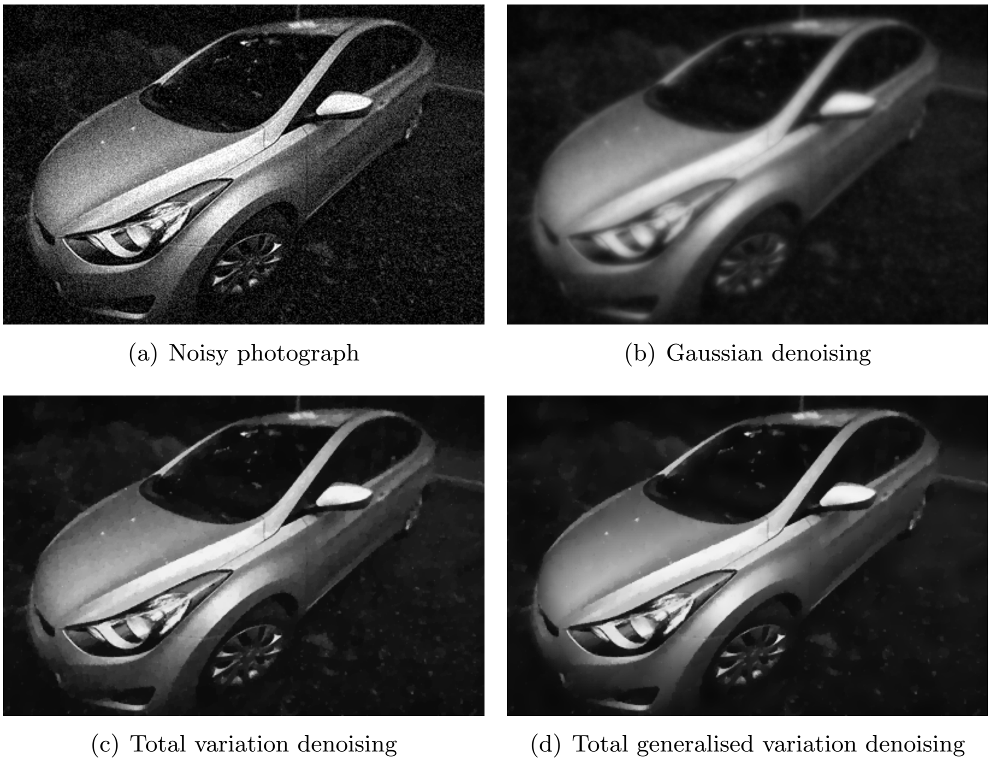
[1] Computational Science Center, University of Vienna, Nordbergstrasse 15, A-1090 Vienna, Austria
[2] Computational Science Center, University of Vienna, Nordbergstrasse 15, A-1090 Vienna, Austria

Higher-Order Variational Techniques for Image Reconstruction and Enhancement

List of authors:

C.-B. Schönlieb [1]

Restoring the original image contents from distorted measurements is one of the most important tasks in image processing. It comprises the enhancement and reconstruction of images distorted by noise or blur (image denoising/deblurring), the filling-in of gaps in images (image inpainting) and the reconstruction of an image from noisy (and possible undersampled) Fourier/Radon measurements. Within various standard methodologies for the solution of these tasks, variational approaches constitute a rich toolbox of methodologies for image reconstruction and enhancement. These techniques are interesting from both an applicational viewpoint - because they are able to produce qualitatively good visual results and can be captured within automatable processing algorithms - but also from a mathematical analysis point of view - because they show some beautiful mathematical concepts and pose interesting analytical problems.



(a) Noisy photograph

(b) Gaussian denoising

(c) Total variation denoising

(d) Total generalised variation denoising

Figure 1: Different methods for image denoising.[2]

[1]Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK

[2]Photo courtesy of Kostas Papafitsoros

Among the pioneers of these approaches are Rudin, Osher and Fatemi who introduced in 1992 the total variation for image regularisation [8]. In Figure 1c an example for total variation denoising is shown. In comparison with the Gaussian filtered image in Figure 1b image structures, such as edges are much better preserved. Yet, total variation denoising is far from being the 'perfect' denoising method: it introduces the so-called staircasing effect in the parts of the image which undergo a linear change of grey values (such as on the bonnet of the car). In the last couple of years alternatives and extensions of total variation denoising have been proposed, which aim to improve upon the staircasing artefacts by introducing higher-order derivatives into the denoising model, compare e.g. [4, 6] and references therein. A very successful approach along these lines is total generalised variation denoising proposed in [2], compare Figure 1d.

In this presentation we shall concentrate on the specific class of higher-order variational techniques, i.e., second- and third-order. After spending some time on introducing the concept of such methods and giving a historical overview of some important contributions in this area, we will get to know some recently proposed higher-order methods, their mathematical properties and applications. The presentation will be furnished by various numerical examples and applications for image restoration [3, 7], surface interpolation [5] and MRI [1].

References

[1] M. Benning, L. Gladden, D. Holland, C.-B. Schönlieb, and T. Valkonen. Phase reconstruction from sparse mri measurements – a survey of variational approaches. *Preprint*, 2013.

[2] K. Bredies, K. Kunisch, and T. Pock. Total generalized variation. *SIAM J. Imaging Sc.*, 3:492–526, 2011.

[3] M. Burger, L. He, and C.-B. Schönlieb. Cahn-hilliard inpainting and a generalization for grayvalue images. *SIAM Journal on Imaging Sciences*, 2(4):1129–1167, 2009.

[4] A. Chambolle and P.-L. Lions. Image recovery via total variation minimization and related problems. *Numerische Mathematik*, 76:167–188, 1997.

[5] J. Lellmann, J.-M. Morel, and C.-B. Schönlieb. Anisotropic third-order regularization for sparse digital elevation models. *SSVM*, 2013. To appear.

[6] K. Papafitsoros and C.-B. Schönlieb. A combined first and second order variational approach for image reconstruction. *J. Mathematical Imaging & Vision*, 2013. Published online.

[7] K. Papafitsoros, B. Sengul, and C.-B. Schönlieb. Combined first and second order total variation inpainting using split bregman. *IPOL Preprint*, 2012. To appear.

[8] L.I. Rudin, S. Osher, and E. Fatemi. Nonlinear total variation based noise removal algorithms. *Physica D: Nonlinear Phenomena*, 60(1):259–268, 1992.

Motion-Corrected PET Reconstruction

List of authors:

S. Suhr [1]

M. Burger [2]

J. Modersitzki [3]

Motion corrected PET Reconstruction is one of the most challenging tasks in medical imaging. Recently algorithms for reconstruction with incorporated motion informations aswell as algorithms for motion estimation have been developed. We present a variational framework for simultaneous reconstruction and motion estimation with Kullback-Leibler data fidelity term, TV regularization for the tracer activity and hyperelastic regularization for the motion vector field.

We give some details on the modelling and enlighten the difference between describing the activity in Eulerian and Lagrangian coordinates. A First-Discretize-Then-Optimize approach leads to alternating minimizing the funcional in two steps. In the first step we show that we can easily apply standard EM-TV algorithms on the motion corrected case. The second step leads to a standard registration problem, but with a distance measure defined on the detector domain. We use a multi-level approach combined with a modified BFGS method for the optimization. The talk concludes with the comparison of some numerical results.

[1] MIC Lübeck and WWU Münster

[2] WWU Münster

[3] MIC Lübeck

Pontryagin's principle for multidimensional control problems with polyconvex data

List of authors:
M. Wagner [1]

We consider multidimensional control problems of Dieudonné-Rashevsky type

$$F(x,u) \;=\; \int_{\Omega} f(s, x(s), u(s))\, ds \longrightarrow \inf!\,; \quad (x,u) \in W_0^{1,p}(\Omega, \mathbf{R}^n) \,\times\, L^p(\Omega, \mathbf{R}^{nm})\,; \tag{1.1}$$

$$Jx(s) \;=\; \begin{pmatrix} \partial x_1(s)/\partial s_1 & \ldots & \partial x_1(s)/\partial s_m \\ \vdots & & \vdots \\ \partial x_n(s)/\partial s_1 & \ldots & \partial x_n(s)/\partial s_m \end{pmatrix} \;=\; u(s) \quad \text{for almost all } s \in \Omega\,; \tag{1.2}$$

$$u(s) \in \mathrm{K} \subset \mathbf{R}^{nm} \quad \text{for almost all } s \in \Omega \tag{1.3}$$

with n, $m \geq 2$, $\Omega \subset \mathbf{R}^m$ and a compact set $\mathrm{K} \subset \mathbf{R}^{nm}$ with nonempty interior. In the case of a convex integrand $f(s, \xi, \cdot\,)$ and a convex restriction set K, the global minimizers of (1.1) − (1.3) satisfy first-order necessary optimality conditions in the form of Pontryagin's principle even though the usual regularity condition for the equality operator fails. In our talk, we provide extensions of these optimality conditions to the case of *polyconvex* integrands and restriction sets. Applications to hyperelastic image registration will be discussed.

References

[1] Burger, M.; Modersitzki, J.; Ruthotto, L.: *A hyperelastic regularization energy for image registration.* SIAM J. Sci. Comput. **35** (2013), B132 – B148

[2] Dacorogna, B.: *Direct Methods in the Calculus of Variations.* Springer; New York etc. 2008, 2nd ed.

[3] Wagner, M.: *Pontryagin's maximum principle for multidimensional control problems in image processing.* J. Optim. Theory Appl. **140** (2009), 543 – 576

[4] Wagner, M.: *A direct method for the solution of an optimal control problem arising from image registration.* Numerical Algebra, Control and Optimization **2** (2012), 487 – 510

[1] University of Leipzig, Department of Mathematics, P. O. B. 10 09 20, D-04009 Leipzig, Germany.

Minisymposium on Nonsmooth Optimization, Methods and Applications

organized by Joachim Gwinner and Dominikus Noll

A Non-smooth Algorithm and Application to Eigenstructure Assignment

List of authors:

N.M. Dao [1]

D. Noll [2]

P. Apkarian [3]

We discuss a non-smooth algorithm for solving general constrained optimization programs of the form

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & c(x) \leq 0 \\ & Ax \leq b \end{array} \tag{1}$$

where $x \in \mathbb{R}^n$ is the decision variable, and f and c are potentially non-smooth and non-convex, and where the linear constraints are gathered in $Ax \leq b$ and handled directly. We use a progress function at the current iterate x,

$$F(\cdot, x) = \max\{f(\cdot) - f(x) - \mu c(x)_+, c(\cdot) - c(x)_+\},$$

which is successively minimized subject to the linear constraints.

Based on the results on the convergence theory of algorithm discussed in [1, 3], we show the following

Theorem 1. Suppose that f and c in program (1) are lower-C^1 functions such that the following conditions hold.

(a) f is weakly coercive on the constraint set $\Omega = \{x \in \mathbb{R}^n : c(x) \leq 0, Ax \leq b\}$, i.e., if $x^j \in \Omega$ and $\|x^j\| \to \infty$, then $f(x^j)$ is not monotonically decreasing.

(b) c is weakly coercive on $P = \{x \in \mathbb{R}^n : Ax \leq b\}$, i.e., if $x^j \in P$ and $\|x^j\| \to \infty$, then $c(x^j)$ is not monotonically decreasing.

Then the sequence of serious iterates $x^j \in P$ generated by our algorithm is bounded, and every accumulation point x^* of the x^j satisfies $x^* \in P$ and $0 \in \partial_1 F(x^*, x^*) + A^\top \eta(x^*)$, where $\eta(x^*) = \{\eta \in \mathbb{R}^m : \eta_i \geq 0,\ \eta_i = 0 \text{ if } a_i^\top x^* < b_i\}$ with $A := [a_1 \dots a_m]^\top$. In other words, x^* is either a critical point of constraint violation, or a Karush-Kuhn-Tucker point of program (1). $\square$

We next consider the problem of eigenstructure assignment for output feedback control. Our method allows to place the eigenelements (λ_i, v_i, w_i) simultaneously. This is possible by a combination of linear algebra and nonlinear optimization techniques. The efficiency of the new approach is demonstrated for aerospace applications.

[1] Institut de Mathématiques, Université de Toulouse, France and Department of Mathematics and Informatics, Hanoi National University of Education, Vietnam

[2] Institut de Mathématiques, Université de Toulouse, France

[3] ONERA, Department of System Dynamics, Toulouse, France

References

[1] M. Gabarrou, D. Alazard, D. Noll. Design of a flight control architecture using a non-convex bundle method. *Mathematics of Control, Signals, and Systems*, 25 (2): 257–290, 2013.

[2] B.C. Moore. On the flexibility offered by state feedback in multivariable systems beyond closed loop eigenvalue assignment. *IEEE Trans. Automat. Control*, AC-21: 689–692, 1976.

[3] D. Noll. Cutting plane oracles to minimize non-smooth non-convex functions. *Set-Valued and Variational Analysis*, 18 (3–4): 531–568, 2010.

Three-field Modeling of Nonlinear Non-smooth Boundary Value Problems and Differential Mixed Variational Inequalities

List of authors:
Joachim Gwinner [1]

The well-known Babuka-Brezzi theory for mixed variational problems has been extended by Gatica [1, 2] to some classes of variational problems and nonlinear operator equations. This extension leads to three-field variational models that can be understood as dual-dual mixed variational models or as two-fold saddle point formulations. Such augmented variational models are well-adapted for multi-physics problems with different coupled unknown quantities and in particular for mechanical engineering problems, where speaking in terms of solid mechanics, strains and stresses are often of more interest then displacements.

In this talk we discuss how this three-field modeling can be further extended to a class of nonlinear nonsmooth elliptic boundary value problems and also related parabolic initial boundary value problems that stem from steady-state unilateral contact with Tresca friction in solid mechanics and from nonlinear transient heat conduction with unilateral boundary conditions, respectively.

This approach leads to variational inequalities of mixed form for three fields as unknowns. Based on the Browder-Minty monotonicity method for nonlinear variational inequalities we provide a well-posedness result in the elliptic case. Moreover using Mosco set convergence, we also establish stability results, where we admit perturbations in the nonlinear operator and in the nonsmooth functional. Thus we complement earlier stability results in [3, 4] on variational inequalities in the primal form.

References

[1] G. N.Gatica: An application of Babuka-Brezzis theory to a class of variational problems. Appl. Anal. 75(3-4):2000,297-303.

[2] G.N. Gatica: Solvability and Galerkin approximations of a class of nonlinear operator equations. Z. Anal. Anwend. 21(3):2002,761-781.

[3] J. Gwinner: On differential variational inequalities and projected dynamical systems—equivalence and a stability result. Discrete Contin. Dyn. Syst, ser. A , 2007,467–476.

[4] J. Gwinner: Stability of monotone variational inequalities with various applications, Variational inequalities and network equilibrium problems (Erice, 1994), 123–142.

[1] Institut für Mathematik und Rechneranwendung, Universität der Bundeswehr München, Germany

Convergence of non-smooth descent methods and the Kurdyka-Łojasiewicz inequality

List of authors:
Dominikus Noll [1]

It is well-known that gradient-oriented descent methods converge to critical points in the sense of subsequences under the Armijo condition in tandem with a safeguard against too small steps. Absil *et al.* [1] have shown that convergence to a single critical point occurs for analytic objectives. Their proof was later seen to carry over to objective functions satisfying a Kurdyka-Łojasiewicz inequality, see [3]. Genuine new features in the C^1-case were discovered in [2].

In this talk we investigate whether similar convergence results may be expected for non-smooth descent methods. In [4] Attouch and Bolte show indeed that the non-smooth proximal point method converges under the Kurdyka-Łojasiewicz inequality. On the other hand, inspection of [3] seems to indicate that all other known cases of convergence under KL are essentially elaborate combinations of these two prototypes (smooth or proximal point), with basically the same mechanism of proof at work.

In [5] we give strong evidence that there may indeed be a principal reason for this limitation. On the positive side, we also present a class $\mathcal{S}$ of non-smooth functions, containing the upper C^1-functions, which has the property that whenever $f \in \mathcal{S}$ satisfies a KL-inequality, then a gradient-oriented descent scheme for f (suitably generalized to the non-smooth context) converges to a single critical point.

References

[1] P.A. Absil, R. Mahony, B. Andrews. Convergence of the iterates of descent methods for analytic cost functions. SIOPT,16(2):2005,531-547.

[2] D. Noll, A. Rondepierre. Convergence of linesearch and trust region methods using the Kurdyka-Łojasiewicz inequality. Computational and Analytical Mathematics. Springer Proceedings in Mathematics, 2013.

[3] H. Attouch, J. Bolte, B.F. Svaiter. Convergence of descent methods for semi-algebraic and tame problems: proximal algorithms, forward-backward splitting, and regularized Gauss-Seidel methods. Math. Prog. Ser. A, 137(1-2),2013,91-129.

[4] H. Attouch, J. Bolte. On the convergence of the proximal algorithm for non smooth functions involving analytic features. Math. Prog. 116(1-2, ser. B):2009,5-16.

[5] D. Noll. Convergence of non-smooth descent methods using the Kurdyka-Łojasiewicz inequality. Preprint.

[1] Institut de Mathématiques, Université de Toulouse, France

Optimal control of crystallization of α-lactose monohydrate

List of authors:
A. Rachah [1] D. Noll [2] F. Espitalier [3]

Crystallization is a separation technique widely used in the chemical, pharmaceutical, material and semiconductor industries. It is used to produce high purity compounds with tight specifications on product quality. Crystallization of a supersaturated solution is triggered by cooling, evaporation of solvent, addition of anti-solvent, or by chemical reactions [1, 2]. Mathematically, crystallization processes are represented as complex systems based on population, mass or molar, and energy balance equations.

For a number of reasons, there has been growing interest in the crystallization of lactose [3] in the food processing industry, particularly so because α-lactose monohydrate is the most common form of lactose in the fabric of medications. The specificity of crystallization of α-lactose monohydrate is that two forms of lactose (α- and β-lactose) exist simultaneously in aqueous solution, their exchange being governed by mutarotation, and that crystallization of α-lactose is solvated, which means a single water molecule is incorporated into the crystal structure at nucleation. The mathematical model therefore includes four interacting populations, one of them aging, so that controlling the process becomes a challenging task.

In this study we present a model of solvated crystallization of α-lactose monohydrate, and then use it control the process in semi-batch mode by acting on the feed rate, the crystallizer temperature and on the crystal seed as a parameter. The goal is to steer the process in such a way that the growth of small particles within a specified range is privileged. Numerical solutions have been computed with the ACADO [5] toolbox and with the solver PSOPT [4].

References

[1] A.Mersmann : Crystallization Technology Handbook, Marcel Dekker, New York, 2001.

[2] Q. Hu, S.Rohani, D.X. Wang, A. Jutan : Optimal control of a batch cooling seeded crystallizer, Powder Technology, volume 7, 2005.

[3] J. Mcleod : Nucleation and growth of Alpha lactose Monohydrate PhD Thesis, Massey University, volume 220, 2007.

[4] http://www.psopt.org/Home

[5] D. Ariens, B. Houska, H.J. Ferreau : ACADO Toolkit User's Manual. `http://www.acadotoolkit.org.`, 2010.

[1]Université Paul Sabatier, Institut de Mathématiques, Applied Mathematics, Toulouse, France
[2]Université Paul Sabatier, Institut de Mathématiques, Applied Mathematics, Toulouse, France
[3]Ecole des Mines d'Albi, Process Engineering, France

Convergence of Linesearch and Trust-Region Methods using the Kurdyka-Łojasiewicz Inequality

List of authors:
A. Rondepierre [1]

Global convergence for linesearch descent methods traditionally only assures subsequence convergence to critical points, while convergence of the entire sequence of iterates is not guaranteed. Similarly, subsequence convergence in trust-region methods is establish by relating the progress of trial points to the minimal progress achieved by the Cauchy point. These results are usually proved for $C^{1,1}$ or C^2-functions.

Recently Absil *et al.* [1] proved convergence of iterates of descent methods to a single limit-point for analytic objective functions, using the fact that this class satisfies the so called Łojasiewicz inequality. In [6] we prove convergence of linesearch and trust-region descent methods to a single critical point for C^1 functions satisfying the Kurdyka-Łojasiewicz inequality, a generalization of the Łojasiewicz inequality. This is motivated by recent convergence results based on this condition in other fields, see e.g. [2], [4], [5, 3].

For linesearch methods we prove convergence for C^1 functions, and we show that it is allowed to memorize the accepted steplength between serious steps if the objective is of class $C^{1,1}$. This option may be of interest for large scale applications, where second-order steps are not practical, and re-starting each linesearch at $t = 1$ may lead to unnecessary and costly backtracking.

For trust-region methods we discuss acceptance tests which feature new conditions on the curvature of the objective along the proposed step, in tandem with the usual criteria relating the achieved progress to the minimal progress guaranteed by the Cauchy point.

References

[1] P.A. Absil, R. Mahony, B. Andrews. Convergence of the iterates of descent methods for analytic cost functions. SIOPT,16(2):2005,531-547.

[2] H. Attouch, J. Bolte. On the convergence of the proximal algorithm for nonsmooth functions involving analytic features. *Mathematical Programming*, 116(1-2, Ser. B):5–16, 2009.

[3] H. Attouch, J. Bolte, P. Redont, A. Soubeyran. Proximal alternating minimization and projection methods for nonconvex problems: An approach based on the Kurdyka-Łojasiewicz inequality. *Journal Mathematics of Operations Research*, 35(2), 2010.

[4] J. Bolte, A. Daniilidis, A. Lewis, M. Shiota. Clarke subgradients of stratifiable functions. *SIAM Journal on Optimization*, 18(2):556–572 (electronic), 2007.

[1] Institut de Mathématiques, INSA de Toulouse, France

[5] J. Bolte, A. Daniilidis, O. Ley, L. Mazet. Characterizations of Łojasiewicz inequalities: subgradient flows, talweg, convexity. *Transactions of the American Mathematical Society*, 362(6):3319–3363, 2010.

[6] D. Noll, A. Rondepierre. Convergence of linesearch and trust region methods using the Kurdyka-Łojasiewicz inequality. Computational and Analytical Mathematics. Springer Proceedings in Mathematics, 2013.

Efficient Parameter Optimization for Elliptic Homogenization Problems via Model Reduction and Error Control

List of authors:

M. Schaefer [1]

M. Ohlberger [2]

The mathematical description of natural and technical processes often leads to parametrized problems with multiple scales. In applications we often meet the multi-query context: for example in optimal control or during parameter studies, the model has to be evaluated repeatedly for a large variety of different parameters. With classical discretization techniques like FEM this often becomes prohibitively costly. The reduced basis method (RBM) [4] is a well established technique in such scenarios. The general idea behind the RBM is to project the original model onto low-dimensional reduced ansatz spaces. These spaces are built up of sample solutions for specially chosen parameters. The most important ingredient is an *offline/online* decomposition which allows a separation of the high-dimensional computations (e.g. basis generation) from the low-dimensional (parameter dependent) reduced scheme.

In this talk, we are considered with elliptic multiscale problems. We investigate macroscopic cost functionals and treat microscopic design parameters:

$$\left.\begin{array}{rl} \min & J(u^\varepsilon(\mu),\mu) \\ \text{s.t.} & C_j(u^\varepsilon(\mu),\mu) \le 0 \quad \forall j = 1,\dots,N \in \mathbb{N}, \\ & \mu \in \mathcal{P} \subset \mathbb{R}^p \end{array}\right\} \tag{1}$$

with real valued functional J, C_j. Here, $u^\varepsilon(\mu)$ is (weak) solution of the parametrized multiscale problem

$$\left.\begin{array}{c} \nabla \cdot (A^\varepsilon(\mu)\nabla u^\varepsilon(\mu)) = f \\ + \text{ suitable B.C.} \end{array}\right\} \tag{2}$$

with a rapidly oscillating and parameter dependent diffusion tensor $A^\varepsilon(\mu)$.

For the model reduction we replace the multiscale problem by its homogenization limit (cf. [2]) which can be considered as a prototypal approach for more general numerical multiscale schemes that make use of the scale separation (like the heterogeneous multiscale method, cf. [1]). A posteriori error estimates for the forward problem (cf. [3]) are obtained in the periodic homogenization setting using a two scale weak formulation of the original multiscale problem. These error estimates allow for an offline/online splitting and are therefore suitable for both the construction of accurate reduced basis spaces and the online evaluation of the reduced approximations. The method also allows for efficient calculation of parametric derivatives which are neccessary for the optimization algorithm.

[1]Institute for Computational and Applied Mathematics, University of Münster, Einsteinstraße 62, 48149 Münster, Germany (michael.schaefer@uni-muenster.de)

[2]Institute for Computational and Applied Mathematics and Center for Nonlinear Sciences, University of Münster, Einsteinstraße 62, 48149 Münster, Germany (mario.ohlberger@uni-muenster.de)

References

[1] P. Henning, M. Ohlberger (2013): A-posteriori error estimation for a heterogenous multiscale method for monotone operators and beyond a periodic setting. *Accepted for publication in: Discrete and Continuous Dynamical Systems - Series S (DCDS-S)*

[2] M. Ohlberger, M. Schaefer (2012): A reduced basis method for parameter optimization of multiscale problems. *Proceedings of ALGORITMY 2012, Conference of Scientific Computing, Vysoké Tatry – Podbanské, September 9-14, 2012*, pp. 272–281.

[3] M. Ohlberger, M. Schaefer (2013): Error Control Based Model Reduction for Parameter Optimization of Elliptic Homogenization Problems. *Accepted for the Proceedings of the 1st IFAC Workshop on Control of Systems Governed by Partial Differential Equations, Paris, September 25-27, 2013* (accepted).

[4] A.T. Patera, G. Rozza (2007): Reduced Basis Approximation and a Posteriori Error Estimation of Parametrized Partial Differential Equations. *To appear in (tentative rubric) MIT Pappalardo Graduate Monographs in Mechanical Engineering*

Minisymposium on
Optimal Control and Games

organized by Hélène Frankowska and Marc Quincampoix

L^∞ Optimal Control Problems as Dynamic Differential Games

List of authors:

P. Bettiol [1]

F. Rampazzo [2]

L^∞ (or minimax) problems consist in minimizing the (pointwise) maximum of a running cost along the trajectory. These control problems have been extensively studied from various points of view, including dynamic programming, numerical approximations, and necessary conditions (see e.g. [1], [2], [3], [4]). We consider an L^∞ *optimal control problem* in which the payoff is the sum of an L^∞ functional and a classical Mayer functional (the latter being a right end-point functional). Owing to the $\langle L^1, L^\infty\rangle$ duality, we rephrase the L^∞ control problem in terms of a *static differential game*, where a new variable k is introduced and plays the role of an opponent player who wants to *maximize* the cost. A relevant fact is that this static game is equivalent to the corresponding *dynamic differential game*, which allows the (upper) value function to verify a boundary value problem. This boundary value problem involves a Hamilton-Jacobi equation whose Hamiltonian is continuous. The value function of the game $\mathcal{W}(t,x,k)$ –whose restriction to $k=0$ coincides with the value function of the reference L^∞ problem– is continuous and solves the established boundary value problem. Furthermore, $\mathcal{W}$ is the unique viscosity solution in the class of (not necessarily continuous) bounded solutions.

References

[1] Barron E. N., *Viscosity solutions and analysis in L^∞*, Nonlinear Analysis, Differential Equations and Control (Montreal, QC, 1998) (F. Clarke and R. J. Stern, eds.), NATO Sci. Ser. C Math. Phys. Sci., vol. 528, Kluwer Academic Publishers, DorFdrecht, pp. 1-60, 1999.

[2] Barron E. N. and Ishii H., *The The Bellman equation for minimizing the maximum cost*, Nonlinear Anal., Theory Methods Appl., vol. 13, no. 9, pp. 1067–1090, 1989.

[3] Di Marco S. C. and González R.L.V., *Minimax optimal control problems. Numerical analysis of the finite horizon case*, ESAIM: Mathematical Modelling and Numerical Analysis, vol. 33, pp. 23–54, 1999.

[4] Vinter R., *Minimax optimal control*, SIAM J. Control Optim., vol. 44, pp. 939–968, 2005.

[1] Laboratoire de Mathematiques, Université de Bretagne Occidentale, France.

[2] Dipartimento di Matematica, Università di Padova, Italy

An Optimal Control Approach for the Design of Low-energy Low-thrust Trajectories Between Libration Point Orbits

List of authors:

R. Epenoy [1]

In this talk, we investigate the numerical computation of minimum-energy trajectories between Libration point orbits in the circular restricted three-body problem [1]. We will consider a low-thrust spacecraft and will focus our attention on its transfer between Lyapunov orbits around the Lagrange points L_1 and L_2 of the Earth-Moon three-body problem. These departure and arrival periodic orbits will be computed using Lindstedt-Poincaré techniques [2]. It is known from dynamical system theory [3, 4, 5] that almost zero-cost transfers exist for particular values of the transfer duration when the two orbits have the same Jacobi constant. These so-called heteroclinic connections follow in part the invariant manifolds of the departure and arrival orbits [3, 4, 5] and require small impulsive thrusts that are not achievable by means of a low-thrust propulsion system. However, trying to determine low-energy low-thrust trajectories by solving the minimum-energy optimal control problem appears to be very difficult or even impossible from a medium value of the transfer duration. In particular, indirect shooting methods fail mainly due to the hypersensitivity of the state and costate equations.

In this talk, we develop a three-step methodology for solving the minimum-energy optimal control problem without using information from the invariant manifolds of the initial and final orbits. With this aim, we first determine a feasible control with quadratic-zero-quadratic time structure. Then we build an optimal control problem whose solution is equal to this feasible control. In the second step, this problem is embedded in a family of problems depending on a parameter ϵ. For each problem, the departure location from the first orbit and the arrival location at the target orbit are fixed to the non-optimal values associated with the feasible control. These problems are solved by continuation on ϵ until we obtain a suboptimal trajectory connecting the two Lyapunov orbits. Each problem is solved thanks to an indirect single shooting method. The Jacobian of the shooting function is computed using variational equations. Finally, in the last step of the method, the minimum-energy solution is obtained by determining the optimal value of the departure location from the initial orbit and that of the arrival location at the target orbit. Numerical results are provided demonstrating the efficiency of the developed approach for different values of the transfer duration leading to trajectories with one or two revolutions around the Moon.

In conclusion, this talk proposes a new methodology that allows the computation of low-energy low-thrust transfers between Libration point orbits. This methodology, based on indirect optimal control, variational equations and continuation techniques makes no use of invariant manifolds in contrast with existing approaches. The latter either build a non-optimal approximate trajectory based on the invariant manifolds [6, 7] or use direct methods [8] taking advantage of the manifolds. Indeed, direct methods allow enforcing coast arcs when the spacecraft is

[1] Centre National d'Etudes Spatiales, 18 avenue Edouard Belin 31401 Toulouse Cedex 9, France.

supposed to be close to the invariant manifolds, thereby facilitating the convergence towards the low-energy solution [9]. The methodology presented in this talk paves the way for the development of an effective indirect approach for computing low-energy low-thrust Earth-Moon transfers in the Sun-Earth-Moon bicircular restricted four body problem. This challenging problem of great interest has only been addressed for far by means of direct methods [10].

References

[1] V. G. Szebehely. Theory of Orbits - The Restricted Problem of Three Bodies. Academic Press Inc., Harcourt Brace Jovanovich Publishers, Orlando, Florida, 1967, pp. 8-100.

[2] J. Masdemont. High Order Expansions of Invariant Manifolds of Libration Point Orbits with Applications to Mission Design. *Dynamical Systems*, 20:1, 2005, pp. 59-113.

[3] W. S. Koon, M. W. Lo, J. E. Marsden, and S. D. Ross. Heteroclinic Connections Between Periodic Orbits and Resonance Transitions in Celestial Mechanics. *Chaos*, 10:4, 2000, pp. 427-469.

[4] G. Gómez, W. S. Koon, J. E. Marsden, J. Masdemont, and S. D. Ross. Connecting Orbits and Invariant Manifolds in the Spatial Restricted Three-body Problem. *Nonlinearity*, 17:5, 2004, pp. 1571-1606.

[5] E. Canalias and J. Masdemont. Homoclinic and Heteroclinic Transfer Trajectories Between Lyapunov Orbits in the Sun-Earth and Earth-Moon Systems. *Discrete and Continuous Dynamical Systems - Series A*, 14:2, 2006, pp. 261-279.

[6] Y. Ren, P. Pergola, E. Fantino, and B. Thiere. Optimal Low-thrust Transfers Between Libration Point Orbits. *Celestial Mechanics and Dynamical Astronomy*, 112:1, 2012, pp. 1-21.

[7] P. Pergola, K. Geurts, C. Casaregola, and M. Andrenucci. Earth-Mars Halo to Halo Low Thrust Manifold Transfers. *Celestial Mechanics and Dynamical Astronomy*, 105:1-3, 2009, pp. 19-32.

[8] B. A. Conway. A Survey of Methods Available for the Numerical Optimization of Continuous Dynamic Systems. *Journal of Optimization Theory and Application*, 152:2, 2012, pp. 271-306.

[9] M. T. Ozimek and K. C. Howell. Low-Thrust Transfers in the Earth-Moon System, Including Applications to Libration Point Orbits. *Journal of Guidance, Control and Dynamics*, 33:2, 2010, pp. 533-549.

[10] G. Mingotti, F. Topputo, and F. Bernelli-Zazzera. Efficient Invariant-manifold, Low-thrust Planar Trajectories to the Moon. *Communications in Nonlinear Science and Numerical Simulation*, 17:2, 2012, pp. 817-831.

A Second-Order Maximum Principle in Optimal Control under State Constraints

List of authors:

H. Frankowska [1]

D. Hoehener [2]

D. Tonon [3]

This talk will address new second order necessary optimality conditions for the Mayer optimal control problem under state constraints. The obtained conditions include both pointwise and integral inequalities and extend earlier results.

For this aim we first derive a second-order variational inclusion for control systems under state constraints and obtain a sufficient condition for normality of the maximum principle. Then we apply these results to get a second order necessary optimality condition, which leads to a maximum principle under state constraints containing three additional inequalities. The first one involves the second order tangent to the dynamics, while the second one involves the derivatives of dynamics with respect to the state variable. Both these inequalities have to be satisfied pointwise. Finally the third inequality is of the integral type, like the classical one in the second order necessary optimality conditions, but it may contain extra terms.

Some extensions to the Mayer differential inclusion problem under state constraints will be also provided. Details can be found in [2].

References

[1] A. Cernea and H. Frankowska, *A connection between the maximum principle and dynamic programming for constrained control problems*, SIAM J. Control Optim., 44 (2005), pp. 673–703.

[2] H. Frankowska, D. Hoehener and D. Tonon, *A Second-Order Maximum Principle in Optimal Control under State Constraints*, submitted.

[3] H. Frankowska, D. Tonon, *Inward Pointing Trajectories, Normality of the Maximum Principle and the non Occurrence of the Lavrentieff Phenomenon in Optimal Control under State Constraints*, J. Conv. Anal., 20 (4) (2013).

[4] D. Hoehener, *Variational approach to second-order optimality conditions for control problems with pure state constraints*, SIAM J. Control Optim. 50 (2012), 1139-1173.

[5] R. B. Vinter, Optimal Control, Birkhaüser, Boston, 2000.

[1] CNRS and Université Pierre et Marie Curie (Paris 6), France

[2] Université Pierre et Marie Curie (Paris 6), France

[3] Imperial College, London, United Kingdom

Second-Order Necessary and Sufficient Optimality Conditions for Constrained Control Problems

List of authors:

D. Hoehener [1]

We consider an optimal control problem of the Mayer form with the control system $\dot{x} = f(t, x, u)$, control constraints $u(t) \in U(t)$ and pure state constraints $x(t) \in K$.

Traditionally, second-order optimality conditions for optimal control problems with state constraints are derived from abstract results stated for mathematical programming problems in infinite dimensional Banach spaces. In this presentation we use instead a variational approach and show how to obtain second-order optimality conditions directly, by using second-order variational equations.

The main advantage of such direct approach is that we are no longer bound to the Banach space setting, therefore we are able to prove optimality conditions for optimal controls that are merely measurable in contrast to many known results that require optimal controls to be continuous or piecewise continuous. In addition, it allows to separate the proofs of the first- and second-order necessary conditions. In particular, our result applies to any first-order necessary optimality conditions in the form of the constrained maximum principle. Another important outcome of this approach is that we have to impose only very mild assumptions on the control constraints and in the case of necessary conditions, even the set of state constraints K can be general.

[1] Combinatoire & Optimisation, Institut de Mathématiques de Jussieu (UMR 7586), Université Pierre et Marie Curie, 4 place Jussieu, 75252 Paris cedex 05, France

Energy-efficient optimization solvers with sub-microsecond latencies

List of authors:

E.C. Kerrigan [1] J.L. Jerez [2] G.A. Constantinides [3] S. Richter [4] P.J. Goulart [5] M. Morari [6]

A microprocessor can perform at least one order of magnitude more mathematical operations per second per watt by using fixed-point, rather than floating-point arithmetic units. The main problem in implementing an algorithm in fixed-point is to decide how many bits to assign to the integer part in order to avoid overflow errors and/or how to modify an algorithm so that overflow cannot occur. We will show how to solve this problem and provide a priori theoretical guarantees that overflow will not occur for two important algorithms:

(i) The *Lanczos algorithm* is the fundamental building block of the most widely used iterative linear solvers, such as the conjugate gradient and minimal residual method. By modifying the algorithm and scaling the data in a suitably-defined manner, overflow errors can be avoided. We will demonstrate the successful implementation of our results within an interior point solver, which computes optimal input trajectories for a real-time flight control application. Our fixed-point implementation of the Lanczos algorithm can sustain more than 40 billion operations per second per watt on current embedded processors, while still achieving the same accuracy as a double-precision floating-point implementation. This compares favorably to the Nvidia GeForce GTX 690, which has a specification of 18.74 billion floating-point operations per second (gigaflops) per watt, and the Beacon-Appro GreenBlade GB824M, the most energy efficient supercomputer on the Green500 list (www.green500.org) of November 2012, which is capable of 2.5 gigaflops per watt when running the LINPACK benchmark.

(ii) The *fast gradient method* of Nesterov has attracted considerable attention over the last few years, due to its ease of implementation and good performance on a large class of problems. This method is particularly amenable to analysis under the assumption of fixed-point arithmetic. We will present theoretical results that can be used to determine a priori the number of bits required to achieve a given accuracy in the solution. We will demonstrate how we have successfully used these results to implement a predictive controller for an atomic force microscope on a low-end processor, while achieving control update rates in excess of 1 MHz.

The ability to be able to successfully implement linear algebra solvers and optimization algorithms in fixed-point arithmetic, with suitable theoretical guarantees, opens up the possibility of using optimization methods to solve control and signal processing problems in application areas that have been considered too challenging up to now.

[1] Departments of Electrical & Electronic Engineering and Aeronautics, Imperial College London, UK
[2] Department of Electrical & Electronic Engineering, Imperial College London, UK
[3] Department of Electrical & Electronic Engineering, Imperial College London, UK
[4] Automatic Control Laboratory, ETH Zürich, Switzerland
[5] Automatic Control Laboratory, ETH Zürich, Switzerland
[6] Automatic Control Laboratory, ETH Zürich, Switzerland

Value of a Differential Game without Isaacs' condition

List of authors:
J.P. Maldonado-López [1]

We consider a zero-sum differential game where players make simultaneous moves (chosen by randomization) on the nodes of a commonly known time partition. This allows us to avoid Isaacs' condition. The limit as the mesh of the partition goes to zero is characterized as the viscosity solution of an appropriate HJI equation. Our work builds on a result of W.H.Fleming [3]. As an application, we provide a model for a continuous time stochastic game.

References

[1] Rainer Buckdahn, Juan Li, and Marc Quincampoix Value function of differential games without isaacs conditions. an approach with nonanticipative mixed strategies *International Journal of Game Theory*, 1-32, 2012.

[2] Pierre Cardaliaguet, Rida Laraki, and Sylvain Sorin A continuous time approach for the asymptotic value in two-person zero-sum repeated games *SIAM J. Control Optim.*, 50, no. 3, 1573-1596, 2012.

[3] W. H. Fleming note on differential games of prescribed duration, Contributions to the theory of games, vol. 3, A *Annals of Mathematics Studies*, Princeton University Press, Princeton, N.J., 1957, pp. 407412.

[1] Université Pierre et Marie Curie, Paris, France.

Optimal Control of a SEIR Epidemic Model with Mixed and State Constraints

List of authors:

Maria do Rosario de Pinho [1]

Md. Haider Ali Biswas [2] Igor Kornienko [3]

Luis Tiago Paiva [4]

In [1] Neilan and Lenhart propose an optimal control problem based on a SEIR compartmental model to determine vaccination strategies over a fixed period of time. A particular aspect of this problem is that the optimized criterion is linear with respect to the state and quadratic with respect to the control. Compartmental models are widely used to study vaccination strategies of a certain infectious disease. SEIR models are based on the division of the target population into four compartments; susceptible (S), exposed to the disease but not yet infectious (E), infectious (I), or recovered (immune) (R). Such models can represent many human infectious diseases but in [1] a generic SEIR model is considered.

In this talk we study the introduction of constraints to the problem proposed in [1]. First we add mixed constraints, then we consider pure state constraints and, finally, in the third case, we add mixed constraints to state constraints. Our work can be viewed as a development of [2] where numerical results of these three cases are presented.

As in [2] we use optimal control solvers to determine the numerical solutions of these problems but now we undertake an analytical study of the solutions of the two latter cases. We discuss normality of the solutions and the characterization of some of the multipliers. Moreover, we discuss the applicability of second order sufficient conditions.

References

[1] R. M. Neilan and S. Lenhart, An Introduction to Optimal Control with an Application in Disease Modeling, *DIMACS Series in Discrete Mathematics*, **75** (2010), 67–81.

[2] M.H.A. Biswas, L.T. Paiva and M.d.R. de Pinho, A SEIR model for control of infectious diseases with constraints, submitted to *DIMACS Series in Discrete Mathematics*, 2013.

[1] Universidade do Porto, FEUP, DEEC, ISR, Portugal
[2] Universidade do Porto, FEUP, DEEC, ISR, Portugal
[3] Universidade do Porto, FEUP, DEEC, ISR, Portugal
[4] Universidade do Porto, FEUP, DEEC, ISR, Portugal

Metric Regularity and Stability of Optimal Control Problems for Linear Systems

List of authors:

M. Quincampoix [1]

V. Veliov [2]

We investigate stability properties of the solutions of optimal control problems for linear systems. The analysis is based on an adapted concept of metric regularity, the so-called *strong bi-metric regularity*, which is introduced and investigated in the paper. It allows to give a more precise description of the effect of perturbations on the optimal solutions in terms of a Hölder-type estimation, and to investigate the robustness of this estimation. The Hölder exponent depends on a natural number k, which is known as the *controllability index* of the reference solution. An inverse function theorem for strongly bi-metrically regular mappings is obtained, which is used in the case $k = 1$ for proving stability of the solution of the considered optimal control problem under small non-linear perturbations. Moreover, a new stability result with respect to perturbations in the matrices of the system is proved in the general case $k \geq 1$.

References

[1] H. Frankowska, M. Quincampoix. Hölder metric regularity for set-valued maps. *Math. programming, Ser. A*, **132**(1):333-354, 2012.

[2] M. Quincampoix, V. Veliov, Metric Regularity and Stability of Optimal Control Problems for Linear Systems, submitted

[1] Laboratoire de Mathématiques, Université de Brest, France,

[2] Institute of Mathematical Methods in Economics, Vienna University of Technology, Austria,

Proximal subdifferentiability and local regularity of the value function of Mayer's problem with differential inclusions

List of authors:

T. Scarinci [1]

The goal of this talk is the study of the local regularity of the value function of a Mayer problem, where the state equation is given by a differential inclusion of the type:

$$\begin{aligned} &\dot{x}(t) \in F(x(t)), \text{ for a.e. } t \in [t_0, T], \\ &x(t_0) = x_0, \end{aligned} \tag{1}$$

where F is a multifunction subject to suitable structural assumptions. The value function of such a problem is locally semiconcave in $(-\infty, T] \times \mathbb{R}^n$ (see [2]) and satisfies, in the viscosity sense, the Hamilton Jacobi equation:

$$-u_t + H(x, -u_x) = 0, \tag{2}$$

where H is the Hamiltonian defined by:

$$H(x,p) = \sup_{v \in F(x)} \langle v, p \rangle. \tag{3}$$

On the other hand, it is well known that V fails to be everywhere differentiable, in general. Even when V is differentiable at a point (t, x), this dress not yields that V is smooth in a neighborhood of (t, x). When equation (2) is associated to a Bolza problem in the calculus of variations, a sufficient condition for the local regularity of V has been recently proposed in [1] . Such a condition requires, among others things, the existence of a proximal subgradient of V at (t_0, x_0).

In this talk we will recover a similar result for a Mayer problem associated to (1). An essential step of the analogies is the proof of a so-called *proximal subgradient inclusion*, which is also a result of indipendent interest. The main technical difficulty of the recent problem, compared with the situation studied in [1] , is the fact that the Hassian matrix is only positively semidefinite in the case of Mayer's problem. Indeed, for a geometric Hamiltoniana we have that:

$$H_{pp}(x,p)p = 0, \text{ for all } x \in \mathbb{R}^n, p \in \mathbb{R}^n \setminus \{0\}. \tag{4}$$

Nevertheless, we will prove that if $H_{pp}(x, p)$ is positive definite in all direction that are orthogornal to a vector p, then we can derive the local smoothness of V.

References

[1] P. Cannarsa, H. Frankowska, *From pointwise to local regularity for solutions of Hamilton-Jacobi-Bellamn equations*, to appear in Calculus Variation PDE.

[1] University of Rome Tor Vergata

[2] P. Cannarsa, P. Wolenski, *Semiconcavity of the value function for a class of differential inclusion*, Discrete and Continuous Dynamical Systems, Volume 29, Number 2, February 2011, pp 453-466.

Stability of value functions for state constrained Bolza problems.

List of authors:

H. Sedrakyan [1]

In this talk we consider a family of Bolza optimal control problems and investigate stability properties of their value functions. The stability is guaranteed by the classical assumptions imposed on Hamiltionians and an inward pointing condition on state constraints. As a biproduct of this investigation we also show uniqueness of solutions to a family of state-constrained Hamilton-Jacobi equations and new representation theorems for Hamiltonians that are convex in the last variable.

References

[1] Aubin J.P., Frankowska H. Set-valued analysis. *Birkhauser*, 1990.

[2] Frankowska H., Bettiol P., Vinter R. L^∞ estimates on trajectories confined to a closed subset. *Journal of differential equations*, 1912–1933, 2012.

[3] Frankowska H., Mazzola M. Discontinuous solutions of Hamilton-Jacobi-Bellman equation under state constraints. *Calculus of Variations, Springer-Verlag*, 2012.

[4] Rampazzo F. Faithful representations for convex Hamilton-Jacobi equations. *SIAM Journal Control Optimal*, 867–884, 2005.

[5] Rockafellar T., Wets R. Variational analysis *Springer*, 1998.

[1]Université Pierre et Marie Curie, Paris, France.

On some links between discrete time and continuous time dynamic games

List of authors:

S. Sorin [1]

We will describe some connections between two-person zero-sum games in discrete and continuous time.

The first topic concerns approachability. We show the relation between weak approachability (asymptotic approach) in discrete time repeated games and the value of differential games with fixed duration. Similarly we exhibit the connection, in terms of value and strategies, between approachability (uniform approach) in repeated games and qualitative differential games.

A second topic is the use of comparison theorems and viscosity tools to prove the existence of an asymptotic value, for all vanishing evaluations in three classes: games with incomplete information, absorbing games and splitting games.

References

[1] Assoulamani S., M. Quincampoix and S. Sorin (2009) Repeated games and qualitative differential games: approachability and comparison of strategies, *SIAM Journal on Control and Optimization*, **48**, 2461-2479.

[2] P. Cardaliaguet, R. Laraki and S. Sorin, A Continuous Time Approach for the Asymptotic Value in Two-Person Zero-Sum Repeated Games. *SIAM Journal on Control and Optimization* **50** (2012), 1573-1596.

[1] Faculté de Mathmatiques, Université Pierre et Marie Curie - Paris 6

The Goh and the generalized Legendre-Clebsch conditions for closed control constraints

List of authors:

D. Tonon [1]

H. Frankowska [2]

We present second order necessary optimality conditions for the Mayer optimal control problem when the optimal control $\bar{u}(\cdot)$ is singular. Indeed, in this case, the classical Legendre-Clebsch condition is no more a useful necessary optimality condition. Alternative pointwise necessary optimality conditions, known as Goh and generalized Legendre-Clebsch conditions, were proved by Goh in [3], in the case of an open control constraint. When the control set U is a closed subset of $\mathcal{R}^m$, new techniques are necessary even when dealing with points that are in the interior of it. We show that, if an optimal control $\bar{u}(\cdot)$ is singular and integrable, then for almost every t such that $\bar{u}(t)$ is in the interior of U, both the Goh and a generalized Legendre-Clebsch conditions hold true. Moreover, when the control set is a convex polytope, similar conditions are verified on the tangent subspace to U at $\bar{u}(t)$ for almost all t's such that $\bar{u}(t)$ lies on the boundary ∂U of U. The Goh condition is valid also for U having a smooth boundary at every t where $\bar{u}(\cdot)$ is singular, integrable and $\bar{u}(t) \in \partial U$. These conditions are contained in [1, 2].

References

[1] H. Frankowska, D. Tonon, Pointwise second-order necessary optimality conditions for the Mayer problem with control constraints, submitted 2013.

[2] H. Frankowska, D. Tonon, The Goh necessary optimality conditions for the Mayer problem with control constraints, submitted to proceedings of the CDC conference 2013.

[3] Goh, B.S. Necessary conditions for singular extremals involving multiple control variables. *SIAM J. Control*, 4:716–731, 1966.

[1] CEREMADE, Paris Dauphine

[2] Institut de Mathématiques de Jussieu, UPMC Paris 6

Regular papers

On the criticality for vector-valued functions

List of authors:
Ewa M. Bednarczuk [1]

In this talk we analyse the criticality for vector-valued functions basing ourselves on the modification of the definition given by Smale. The proposed approach relies strongly on quasi-relative interior of the ordering cone instead of its topological interior. This allows us to make the concept of criticality operational for vector optimization problems where the ordering cone has empty topological interior. With the help of the introduced concept we prove necessary optimality conditions for closed convex pointed cones (with nonempty quasi-relative interiors) and cone-convex vector-valued functions as well as for closed convex pointed generating cones and general directionally differentiable vector-valued mappings.

[1]System Research Institute, Polish Academy of Sciences, ul. Newelska 6, 01-447 Warszawa, Poland

Proximal alternating minimization methods

List of authors:

Jérôme Bolte [1]

joint work with M. Teboulle and S. Sabach

We introduce a proximal alternating linearized minimization (PALM) algorithm for solving a broad class of nonconvex and nonsmooth minimization problems. Building on the powerful Kurdyka-Lojasiewicz property, we derive a self-contained convergence analysis framework and establish that each bounded sequence generated by PALM globally converges to a critical point. Our approach allows to analyze various classes of nonconvex-nonsmooth problems and related nonconvex proximal forward-backward algorithms with semi-algebraic problem's data, the later property being shared by many functions arising in wide variety of fundamental applications. A by-product of our framework also shows that our results are new even in the convex setting. As an illustration of the results, we derive a new and simple globally convergent algorithm for solving the sparse nonnegative matrix factorization problem.

[1] Université Toulouse Capitole - Toulouse School of Economics, Manufacture des Tabacs, 21 Allée de Brienne, 31015 Toulouse, France

Error Bounds for a Discrete Optimal Control Problem with State Constraints

List of authors:

J. Frédéric Bonnans [1]

Adriano Festa [2]

We study the error introduced in the solution of an optimal control problem with state constraints, for which the trajectories are approximated with a classic Euler scheme. Although some results in the subject have been already presented by Hager and Dontchev [3] our analysis is proposed here as an another point of view on the issue. We derive our results using some techniques coming from the sector of perturbation analysis [2] and that procedure has some interesting and original aspects.

The problem can be presented in the following way: let us consider the following state constrained optimal control problem

$$(\mathcal{P}) \begin{cases} \text{Minimize } \phi(y(T)); & \text{subject to} \\ \dot{y}(t) = f(u(t), y(t)), & \text{for a.a. } t \in [0,T]; \\ y(0) = y_0; & \\ g(y_t) \leq 0 & t \in [0,T]. \end{cases} \tag{1}$$

where the control $u(t)$ and the state $y(t)$ are respectively in the spaces $\mathcal{U} := L^\infty(0,T;\mathbb{R}^m)$ and $\mathcal{Y} := W^{1,\infty}(0,T;\mathbb{R}^n)$, and $g : \mathbb{R}^n \to \mathbb{R}^m$ is the constraint. We call $u^*(t)$ an optimal control which generates an optimal trajectory for the problem. We consider now, the Euler discrete version of the same problem, where an uniform sampling of the time is introduced, i.e. for $N \in \mathbb{N}$ and $h = 1/N$ we call $t_k := kh$ for $k = 0, ...N$ and $u_k := u(t_k)$, $y_k := y(t_k)$.

$$(\mathcal{P}_d) \begin{cases} \text{Minimize } \phi(y_N); & \text{subject to} \\ y_{k+1} = y_k + hf(u_k, y_k), & \text{for } k = 0, ..., N-1; \\ y_0 = y_0; & \\ g(y_k) \leq 0 & \text{for } k = 0, ..., N-1. \end{cases} \tag{2}$$

In this work, we arrive to prove that the error $||u^*(t_k) - u_k||_\infty$ is an $O(h)$. Analogous rates will be obtained also for the costate function and the constraints multiplicators. Some peculiar difficulties due to the state constraints will be discussed and studied.

The interest in this subject is widely justified by the large use of this kind of numerical schemes in applications.

[1] INRIA Saclay, *Frederic.Bonnans@inria.fr*

[2] Imperial College London, *a.festa@imperial.ac.uk*

References

[1] J.F. Bonnans, J. Laurent-Varin, Computation of order conditions for symplectic partitioned Runge-Kutta schemes with application to optimal control. *Num. Math.* 103 (2006), 1–10.

[2] J.F. Bonnans and A. Shapiro, *Perturbation analysis of optimization problems.* Springer, New York, 2000.

[3] A.L. Dontchev, W.W. Hager, The Euler approximation in state constrained optimal control. *Math. Comp.* 70 (2001), 173–203.

[4] A.L. Dontchev, W.W. Hager, K. Malanowski Error bounds for Euler approximation of a state and control constrained optimal control problem. *Numer. Funct. Anal. Optim.* 21 (2000), 653–682.

[5] V.M. Veliov, Error analysis of discrete approximations to bang-bang optimal control problems: the linear case. *Control Cyb.* 34 (2005), 967–982.

How to Deal with the Pareto set of a Multiobjective Optimal Control of Parabolic Systems

List of authors:

H. Bonnel [1]

The solution set (Pareto or efficient set) of a multiobjective optimization problem is often very large (infinite and even unbounded). The grand coalition of a cooperative differential game can be written as a multiobjective optimal control problem. Assuming that this game is supervised by a decision maker (DM), the DM can use his own (scalar) objective for choosing a solution (control). Of course this solution must satisfy all the players of the grand coalition, hence must be a Pareto solution. Another interest for the study of this problem is that it may be possible to avoid the generation of all the Pareto controls set.

For multiobjective mathematical programming problems (finite dimensional optimization) there are many contributions in this field (see e.g. [5] for an extensive bibliography). Some recent results for the stochastic case can be found in [2]. My talk deals with a new setting: mulitobjective control of convex optimal parabolic systems, generalizing some results of [4], which have been extended for semivectorial bilevel optimization problems in [3]. My talk is based on my recent paper [1].

References

[1] Henri Bonnel. Post-Pareto Analysis for Multiobjective Parabolic Control Systems. *Ann. Acad. Rom. Sci. Ser. Math. Appl.*, 5: 13–34, 2013.

[2] H. Bonnel and J. Collonge. Stochastic Optimization over a Pareto Set Associated with a Stochastic Multi-objective Optimization Problem. *Journal of Optimization Theory and Applications,* (in press).

[3] H. Bonnel and J. Morgan. Semivectorial Bilevel Convex Optimal Control Problems: An Existence Result. *SIAM Journal on Control and Optimization*, 50, (6): 3224–3241, 2012.

[4] H. Bonnel and Y. Kaya. Optimization Over the Efficient Set of Multi-objective Convex Optimal Control Problems. *Journal of Optimization Theory and Applications,* 147, (1): 93–112, 2010.

[5] Y. Yamamoto. Optimization over the efficient set : an overview. *J. Global Optim.*, 22: 285-317, 2002.

[1] ERIM, University of New Caledonia, B.P. R4, F98851 Nouma Cedex, France, (henri.bonnel@univ-nc.nc)

The application of advanced optimization techniques for fast identification of step changes in parameters of linear continuous system

List of authors:

W. Byrski [1] J. Byrski [2] D. Kmiecik [3] Ł. Kuśnierz [4]

A main contribution of this talk will be the presentation and wide discussion on the new advanced optimization methodologies used for fast identification of abrupt changes of parameters in continuous systems and the application of these algorithms to identification of parameters of real physical system represented by the model of *Segway Personal Transporter* (inverted pendulum). Researches on the efficient and effective algorithms suitable for this purpose were carried out by the authors[1,2] e.g. in [3, 4]. Variety of other methods for solving this topic are proposed in many publications, but mainly for stochastic signals and discrete systems, e.g. [1]. Identification methods for continuous systems with constant parameters one can find in e.g. works of H. Unbehauen and G.P. Rao. Let us assume that linear, continuous SISO system of n-th order ($m \le n$) is given $\sum_{i=0}^{n} a_i\, y^{(i)}(t) = \sum_{j=0}^{m} b_j\, u^{(j)}(t)$. Signals of system input $u(t)$ and output $y(t)$ are measured for $t \ge 0$, where u and $y \in L^2[0,T]$. The signals $y^{(i)}(t)$, $u^{(j)}(t)$ for $i > 0$, $j > 0$, represent unknown i-th and j-th derivatives of the output and input, respectively, as well as a_i, b_j represent $n+m+2$ unknown constant parameters forming the parameter vector Θ. This model is valid within time interval $t \in [0, t_F]$. At the moment t_F step changes in all parameters will appear, hence for $t > t_F$ the new model with another unknown parameters $\bar{\Theta}$ should be assumed $\sum_{i=0}^{n} \bar{a}_i\, \bar{y}^{(i)}(t) = \sum_{j=0}^{m} \bar{b}_j\, \bar{u}^{(j)}(t)$. Before starting the main computation stage of parameter identification, one should solve the problem of unknown derivatives by the use of some type of preprocessing calculations based on convolution transformation of the differential models into their algebraic forms with the same unknown parameters. This procedure is known as modulating function method [5, 6]. Special modulating function φ with compact support and known derivatives $\varphi^{(i)}$ is used. Such transformation represents the finite interval moving window of width h, continuously shifting along the time axis. This operation generates the new functions $y_i(t)$, $u_i(t)$ and $\bar{y}_i(t)$, $\bar{u}_j(t)$ and forms two algebraic models with parameters Θ and $\bar{\Theta}$:

$$\sum_{i=0}^{n} a_i\, y_i(t) = \sum_{j=0}^{m} b_j\, u_j(t) \quad t \in [h, t_F] \qquad \textit{and} \qquad \sum_{i=0}^{n} \bar{a}_i\, \bar{y}_i(t) = \sum_{j=0}^{m} \bar{b}_j\, \bar{u}_j(t) \quad t \in [t_F + h, T]$$

For the best identification of each algebraic model in some chosen interval T_{ID} the problem of minimization of the norm of equation error function $\varepsilon_{EE}(t)$ in $L^2[h, h+T_{ID}]$ will be solved. This norm is represented by quadratic form of parameters Θ with real symmetric Gram matrix $\mathbf{G}$ (or $\bar{\mathbf{G}}$) of inner products of all functions $y_i(t)$, $u_i(t)$ (or $\bar{y}_i(t)$, $\bar{u}_i(t)$), for both models. The solution

[1]Department of Automatics and Biomedical Engineering, DABE, AGH University of Science and Technology
[2]Department of Applied Computer Science, AGH University of Science and Technology
[3]DABE, AGH University of Science and Technology, PhD Student
[4]DABE, AGH University of Science and Technology, PhD Student

of optimization tasks goes from the minimization of these norms under quadratic constraints (parameters belong to unit balls). The optimal parameters are represented by an eigenvector w_{min} which corresponds to the minimal eigenvalue λ_{min} of suitable Gram matrix. The same optimization method one can apply in on-line mode with the use of the Moving Identification Window and the sliding interval $[t - T_{ID}, t]$. This method of identification for constant or slow varying parameters gives very good results and was presented and tested by the authors[1,2] in different publications e.g. [2, 3]. However, for rapid changes of parameters it occurred that after the moment of fault t_F this method gives bad results in the interval $t \in [t_F, t_F + h + T_{ID}]$. This is due to the fact that in this interval the preprocessing convolution procedure is not valid on the whole window $[t - h, t]$, because it will depend partly on Θ and partly on $\bar{\Theta}$. In work [4] the authors proposed the solution to this problem by the formulation of the new rules for calculation of the convolution transformation inside the interval $t \in [t_F, t_F + h]$, by dividing the convolution window $[t - h, t]$ into two subintervals: $[t - h, t_F]$ and $[t_F, t]$, connected with Θ and $\bar{\Theta}$, respectively. Consequently, in the second stage of identification, it will be possible to use another type of the main identification processing window with expanding width $[t_F, t]$ for every t, instead of the standard moving identification window of fixed width $[t - T_{ID}, t]$. It will enable identification of new parameters $\bar{\Theta}$ for $t > t_F$ practically without any dead time. Hence, for the interval $t \in [t_F, t_F + h]$ the new formula for calculation of the optimal parameters was derived and has a form of the continuous version of the least squares optimal solution for parameter identification. The numerical experiments have confirmed fast and accurate properties of the new identification idea. The results of identification of inverted pendulum parameters will be presented in final version of the paper.

References

[1] M. Basseville, I.V. Nikiforov, Detection of Abrupt Changes:Theory and Application, *Prentice-Hall*, 1993.

[2] W. Byrski, S. Fuksa, Optimal identification of continuous systems in L^2 space by the use of compact support filter, *International Journal of Modelling & Simulation*, 15(4), 1995.

[3] W. Byrski, J. Byrski, The role of parameters constrain in EE and OE methods for optimal identification of continuous models, *Journal of Appl. Mathematics and Computer Science*, no.2, 2012.

[4] J. Byrski, W. Byrski, Implementation of a New Algorithm for Fast Diagnosis of Step Changes in Parameters of Continuous Systems, *8th IFAC Symposium on Fault Detection, SAFEPROCESS2012*, August 2012, Mexico City.

[5] V. Maletinsky, Identification of continuous dynamical systems with spline-type modulating functions method, *Proc. of V IFAC Symp. on Ident. and SPE*, Darmstadt, 1979, vol.1.

[6] H.A. Preisig, D.W.T. Rippin, Theory and application of the modulating function method, *Computers Chem.. Engng*, 17(1), Pergamon Press, 1993, 1-40.

Second-Order Necessary Conditions in Pontryagin Form for Optimal Control Problems

List of authors:
X. Dupuis [1]
J. F. Bonnans [2] L. Pfeiffer [3]

We say that optimality conditions for an optimal control problem are in Pontryagin form if they only involve Lagrange multipliers for which Pontryagin's minimum principle holds. This restriction to a subset of multipliers is a strengthening for necessary conditions, and enables sufficient conditions to give strong local minima [4]. We consider optimal control problems with pure state and mixed control-state constraints, and we present first- and second-order necessary conditions in Pontryagin form [1].

They are obtained by a technique of partial relaxation, based on the sliding modes introduced by Gamkrelidze. The partial relaxation furnishes a sequence of auxiliary optimal control problems, for which a Pontryagin minimum of the original problem is a weak minimum; necessary conditions for this weak minimum appear, at the limit, to be in Pontryagin form for the Pontryagin minimum. This technique is classically used to prove Pontryagin's principle, i.e. first-order necessary conditions, for general optimal control problems [3]. It has also been combined with the theory of γ-conditions, whereas we apply here standard optimality conditions, to derive strengthened second-order necessary conditions [5]. The quadratic conditions in [5] are obtained for problems without pure state constraints; our conditions in Pontryagin form do not take into account broken extremals.

We mention that we are also able to provide second-order sufficient conditions in Pontrygin form, and thus to characterize quadratic growth for a strong minimum of such general optimal control problems [2].

References

[1] J. F. Bonnans, X. Dupuis, and L. Pfeiffer. Second-order necessary conditions in Pontryagin form for optimal control problems. Inria Research Report RR-8306, INRIA, May 2013.

[2] J. F. Bonnans, X. Dupuis, and L. Pfeiffer. Second-order sufficient conditions for strong solutions to optimal control problems. Inria Research Report RR-3807, INRIA, May 2013.

[3] A. V. Dmitruk. Maximum principle for the general optimal control problem with phase and regular mixed constraints. *Comput. Math. Model.*, 4(4):364–377, 1993. Software and models of systems analysis. Optimal control of dynamical systems.

[1]CMAP, Ecole Polytechnique & Inria Saclay
[2]CMAP, Ecole Polytechnique & Inria Saclay
[3]CMAP, Ecole Polytechnique & Inria Saclay

[4] A. A. Milyutin and N. P. Osmolovskii. *Calculus of variations and optimal control*, volume 180 of *Translations of Mathematical Monographs*. American Mathematical Society, Providence, RI, 1998.

[5] N.P. Osmolovskii. Necessary quadratic conditions of extremum for discontinuous controls in optimal control problems with mixed constraints. *Journal of Mathematical Sciences*, 183(4):435–576, 2012.

Variational Inequalities in Bang-Singular-Bang Control Investigation

List of authors:

U. Felgenhauer [1]

The talk is devoted to recent results on stability investigation of variational inequalities for bang-singular-bang controls. For the proofs and further explanation we refer to [1, 2].

Consider the following optimal control problem in Mayer form where the system is control-affine, and the data functions smoothly depend on a real parameter p near $p_0 = 0$,

$$\textbf{(CP}_p\textbf{)} \qquad \text{minimize} \quad J_p(x,u) \;:=\; k(x(1),p) \tag{1}$$

subject to

$$\dot{x}(t) \;= f(x(t),p) \,+\, g(x(t),p)\,u(t) \quad \text{a.e. in } [0,1], \tag{2}$$

$$x(0) \;=\; a(p), \tag{3}$$

$$0 \;\le\; u(t) \;\le\; 1, \quad \text{a.e. in } [0,1]\,, \tag{4}$$

$$x \in W^1_\infty(0,1;\mathbb{R}^n), \;\; u \in L_\infty(0,1;\mathbb{R}).$$

The pair $(\bar{x},\bar{u}) \in W^1_\infty \times L_\infty$ is called *admissible* for (CP_p) if (2) – (4) are fulfilled. It will be called a local minimizer (in Pontryagin's sense) if it is admissible, and a constant $\epsilon > 0$ exists such that

$$J_p(\bar{x},\bar{u}) \le J_p(x,u) \quad \text{for all admissible } (x,u) \text{ with } \|x-\bar{x}\|_\infty + \|u-\bar{u}\|_1 < \epsilon.$$

For the given problem, Pontryagin's Maximum Principle yields necessary optimality conditions and will hold in normal form. Denoting by $N_+(\nu)$ the normal cone to the non-negative orthant $\mathbb{R}^n_+$ at $\nu \in \mathbb{R}^n_+$, the conditions can be written in form of a variational inequality

$$\textbf{(VI}_p\textbf{)} \qquad \begin{aligned} &\dot{x} - f(x,p) - g(x,p)u \;=\; 0, \quad x(0) - a(p) \;=\; 0,\\ &\dot{\lambda} + \nabla_x(f(x,p) + g(x,p)u)^T\lambda \;=\; 0, \quad \lambda(1) - \nabla_x k(x(1),p) \;=\; 0,\\ &g(x,p)^T\lambda \,-\, \mu_1 \,+\, \mu_2 \;=\; 0,\\ &-u \;\in\; N_+(\mu_1), \qquad u - 1 \;\in\; N_+(\mu_2) \end{aligned}$$

for almost every $t \in [0,1]$. The function $\lambda \in W^1_\infty(0,1;\mathbb{R}^n)$ is the adjoint or co-state function.

It will be assumed that, for the reference parameter zero, the control has bang-singular-bang structure, i.e., it achieves its extremal values on certain subintervals to the left and right ends of the time interval, and takes "singular" values from the interior of the control set in the remaining part. In [1], the structural stability for bang-singular junction in case of one singular arc of order one was obtained under rather mild assumptions including the strong Legendre condition

[1] Institute of Applied Mathematics and Scientific Computing, Brandenburg University of Technology Cottbus, Germany

but without second-order sufficient optimality conditions. Instead, it had to be supposed that the perturbed system of first-order necessary conditions had a solution.

Recent results on second-order conditions for the problem class in [3] allowed to complete the investigation and to show (i) the existence and local uniqueness of solutions to (VI_p) w.r.t. a L_1 neighborhood of the reference control component, (ii) structural stability together with L_1 Lipschitz continuity of the control w.r.t. parameter perturbation, and (iii) the strict local optimality of the extremal for problem (CP_p). The analysis [2] uses an extended Goh transformation for the linearized form of (VI_p) including the transformation of the related adjoint equation. Under second-order optimality conditions in the spirit of [3], an auxiliary monotone variational inequality is derived which determines the control primitive as an element of L_2.

The main stability proofs given further in [2] are essentially based on S.M. Robinsons work on strongly regular generalized equations. However, the coercivity condition adapted from [3] is too weak for applying the related Lipschitz theory directly. Instead, a first weak continuity result has to be combined with the structural analysis of the underlying controls before final estimates can be obtained.
As a byproduct, structural stability results for a linearization of (VI_p) are provided. Finally, an example illustrates how the assumptions needed for the stability proofs can be checked in simple situation.

References

[1] Structural stability investigation of bang-singular-bang optimal controls. *J. Optim. Theory Appl.*, 152(3):605 - 631, 2012.

[2] Stability analysis of variational inequalities for bang-singular-bang controls. *Control & Cybernetics*, 39 pp., 2013, *to appear.*

[3] S. M. Aronna, J. F. Bonnans, A. V. Dmitruk, P. A. Lotito. Quadratic order conditions for bang-singular extremals *Numer. Algebra, Contr. and Optim.* 2(3):511–546, 2013.

Case Studies for Optimal Control Problems with Delays

List of authors:
L. Göllmann [1]
H. Maurer [2]

In this talk we will present an overview about recent developments in the numerical treatment of optimal control problems with constant time delays:

$$\text{Minimize} \quad J(u,x) = g(x(T))$$

subject to the retarded differential equation, boundary conditions and mixed control-state inequality constraints

$$\begin{aligned}
&\dot{x}(t) = f(t, x(t-r_0), \ldots, x(t-r_d), u(t-r_0), \ldots, u(t-r_d)), \ \text{ a.e. } \ t \in [0,T],\\
&x(t) = x_0(t), x \quad t \in [-r_d, 0],\\
&u(t) = u_0(t), \quad t \in [-r_d, 0),\\
&\psi(x(T)) = 0,\\
&C(t, x(t-r_0), \ldots, x(t-r_d), u(t-r_0), \ldots, u(t-r_d)) \leq 0, \ \text{ a.e. } \ t \in [0,T].
\end{aligned}$$

The functions $g : \mathbb{R}^n \to \mathbb{R}, f : [0,T] \times \mathbb{R}^{(d+1)\cdot n} \times \mathbb{R}^{(d+1)\cdot m} \to \mathbb{R}^n, \psi : \mathbb{R}^n \to \mathbb{R}^q, \quad 0 \leq q \leq n$, and $C : [0,T] \times \mathbb{R}^{(d+1)\cdot n} \times \mathbb{R}^{(d+1)\cdot m} \to \mathbb{R}^p$ are assumed to be continuously differentiable, while the functions $x_0 : [-r_d, 0] \to \mathbb{R}^n, u_0 : [-r_d, 0] \to \mathbb{R}^m$ only need to be continuous.

A minimum principle in form of first-order neccessary optimality conditions for this problem class under consideration of multiple time delays $0 = r_0 < \ldots < r_d$ has been derived in a most recent paper by Göllmann and Maurer [3].

We apply a numerical discretization method by which the delayed control problem is transformed into a nonlinear programming problem. In [3] a proof is given that the associated Lagrange multipliers provide a consistent numerical approximation for the adjoint variables of the delayed optimal control problem. We will illustrate the theory and the numerical approach by various examples taken from chemical engineering and biomedicine.

Among those case studies we will investigate the optimal control of a continuous stirred chemical tank reactor (CSTR) and the optimal treatment of deseases affecting the innate immune response. The CSTR system was studied earlier by Dadebo and Luus [2] using nonlinear programming and dynamic programming methods.

The underlying dynamic model of the innate immune resonse was developed by Asachenkov et al. [1] and Stengel et al. [5, 6]. We will present a multi-drug combination therapy where four control variables are optimized simultaneously. It is obvious to introduce delays in the state and control functions as it usually takes some time for a drug to become effective.

[1] Department of Mechanical Engineering, Münster University of Applied Sciences, Steinfurt, Germany
[2] Institute of Computational and Applied Mathematics, Westfälische Wilhelms-Universität, Münster, Germany

References

[1] A. Asachenkov, G. Marchuk, R. Mohler and S. Zuev, Disease Dynamics. Birkhäuser, Boston, 1994.

[2] S. Dadebo and R. Luus, Optimal control of time-delay systems by dynamic programming. *Optimal Control Applications & Methods*, **13**, 29–41, 1992.

[3] L. Göllmann and H. Maurer, Theory and Applications of Optimal Control Problems with Multiple Time-Delays. *Journal of Industrial an Management Optimization*, to appear.

[4] L. Göllmann, D. Kern and H. Maurer, Optimal control problems with delays in state and control subject to mixed control-state constraints. *Optimal Control Applications and Methods*, **30**, 341–365, 2009.

[5] R. F. Stengel, R. Ghigliazza, N. Kulkarni and O. Laplace, Optimal control of innate immune response. *Optimal Control Applications and Methods*, **23**, 91–104, 2002.

[6] R. F. Stengel and R. Ghigliazza, Stochastic optimal therapy for enhanced immune response. *Mathematical Biosciences*, **191**, 123–142, 2004.

The LQ–problem for infinite–dimensional systems with bounded operators

List of authors:

Piotr Grabowski [1]

Let H, U and Y be Hilbert spaces with scalar products $\langle\cdot,\cdot\rangle_{\mathrm{H}}$, $\langle\cdot,\cdot\rangle_{\mathrm{U}}$ and $\langle\cdot,\cdot\rangle_{\mathrm{Y}}$, respectively. Our aim in this paper is to minimize the quadratic performance index

$$J(x_0,u) =_0^\infty \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}^* \begin{bmatrix} Q & N \\ N^* & R \end{bmatrix} \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \mathrm{d}t \ , \tag{1}$$

where $Q = Q^* \in \mathbf{L}(\mathrm{Y})$, $N \in \mathbf{L}(\mathrm{U},\mathrm{Y})$ and $R = R^* \in \mathbf{L}(\mathrm{U})$, over trajectories of the system

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ x(0) & = & x_0 \\ y(t) & = & Cx(t) \end{array} \right\}, \qquad t \geq 0 \ . \tag{2}$$

where (A,B,C) is a triple of bounded operators $A \in \mathbf{L}(\mathrm{H})$, $B \in \mathbf{L}(\mathrm{U},\mathrm{H})$, $C \in \mathbf{L}(\mathrm{H},\mathrm{Y})$.

The following result, generalizing that of [2], will be proved, discussed and illustrated by an example. In particular, we shall show how this problem is related with an LQ–problem for system having unbounded state, control and observation operators [1]. *If the* observability map*:* $(\Psi x_0)(t) := Ce^{(\cdot)A}x_0$ *belongs to* $\mathbf{L}(H, L^2(0,\infty;Y))$*, the* input-output map $(\mathbb{F}u)(t) := \int_0^t Ce^{(t-\tau)A}Bu(\tau)d\tau$ *is in* $\mathbf{L}(L^2(0,\infty;U), L^2(0,\infty;Y))$ *and there exists* $\varepsilon > 0$ *such that the* Popov spectral function *is coercive, i.e.,* $\Pi(j\omega) := R + 2\,\mathrm{Re}\left[N^*\hat{G}(j\omega)\right] + \left[\hat{G}(j\omega)\right]^* Q\hat{G}(j\omega) \geq \varepsilon I$ *a.e. on j , where(s):=C(sI-A)*$^{-1}$*B, then the problem has a unique solution. The optimal control* $u^c \in L^2(0,\infty;U)$ *can be realized in the linear feedback form*

$$u^c(t) = -R^{-1}\left[B^*\mathcal{H} + N^*C\right]x^c(t) \ , \tag{3}$$

where $\mathcal{H}$ *stands for the minimal cost operator, and* $\mathcal{H}$ *satisfies the* operator Riccati equation

$$A^*\mathcal{H} + \mathcal{H}A + C^*QC = (\mathcal{H}B + C^*N)R^{-1}(B^*\mathcal{H} + N^*C) \ . \tag{4}$$

References

[1] P. Grabowski, *The lq–controller synthesis problem for infinite–dimensional systems in factor form*, OPUSCULA MATHEMATICA, **33** (2013), pp. 29-79.

[2] J.C. Oostveen. R.F. Curtain, *Riccati equations for strongly stabilizable bounded linear systems*, Automatica, **34** (1998), 953-967.

[1] Institute of Control and Biomedical Engineering, AGH University of Science and Technology, A. Mickiewicz avenue 30/B1, rm.314, PL 30-059 Cracow, Poland

Optimal Control Problems for Systems Governed by Evolutionary Inclusions of Second Order

List of authors:

J. F. Han [1]

S. Migórski [2]

In this talk, the optimal control problems for nonlinear second order evolutionary inclusions are investigated. First, the existence of mild solutions (i.e. trajectory–selection pairs) to the inclusion and the continuity properties of the solution set with respect to a parameter are established. Then, for the control problem driven by the evolutionary inclusion, the existence of optimal solutions is shown. Next, allowing the parameter to appear in all the data of the control problem, including the nonlinear operator, the multifunction and the cost functional, the variational stability of control problems is studied. The results on the asymptotic behavior for optimal solutions, the convergence of minimal values, and the stability of reachable sets are delivered. Finally, an example of nonlinear control problem for a hyperbolic problem demonstrates the applicability of the results. More details can be found in [1]–[4].

References

[1] A. Kulig, S. Migorski, Solvability and Continuous Dependence Results for Second Order Nonlinear Evolution Inclusions with a Volterra-type Operator, *Nonlinear Analysis Theory, Methods and Applications* **75** (2012), 4729–4746.

[2] Z. H. Liu, J. F. Han, Boundary Value Problems for Second Order Impulsive Functional Differential Equations, *Dynamic Systems and Applications* **20** (2011), 369–382.

[3] S. Migorski, Existence of Solutions to Nonlinear Second Order Evolution Inclusions without and with Impulses, *Dynamics of Continuous, Discrete and Impulsive Systems, Series B* **18** (2011), 493–520.

[4] S. Migorski, A. Ochal, M. Sofonea, Nonlinear Inclusions and Hemivariational Inequalities. Models and Analysis of Contact Problems, Advances in Mechanics and Mathematics, vol. **26**, Springer, New York, 2013.

[1] Faculty of Mathematics and Computer Science, Jagiellonian University, Institute of Computer Science, ul. Łojasiewicza 6, 30348 Krakow, Poland

[2] Faculty of Mathematics and Computer Science, Jagiellonian University, Institute of Computer Science, ul. Łojasiewicza 6, 30348 Krakow, Poland. The research was supported by the Marie Curie International Research Staff Exchange Scheme Fellowship within the 7th European Community Framework Programme under Grant Agreement No. 295118, the National Science Center of Poland under grant no. N N201 604640, the International Project co-financed by the Ministry of Science and Higher Education of Republic of Poland under grant no. W111/7.PR/2012, the Ministry of Science and Higher Education of Republic of Poland and the Ministry of Science and Technology of the People's Republic of China under the project no. 35-7 within the Bilateral S&T Cooperation Program between Poland and P. R. China for the years 2012–2013, and by the National Science Center of Poland under Maestro Advanced Project no. DEC-2012/06/A/ST1/00262.

From Eckart & Young Approximation To Moreau Envelopes And Vice Versa

List of authors:

Jean-Baptiste Hiriart-Urruty [1]

Hai Yen Le [2]

In matricial analysis, the theorem of Eckart & Young provides a best approximation of an arbitrary matrix by a matrix of rank at most r. In variational analysis or optimization, the Moreau envelopes are appropriate ways of approximating or regularizing the rank function. We prove here that we can go forwards and backwards between the two procedures, thereby showing that they carry essentially the same information.

Keywords. Eckart & Young theorem, Moreau envelopes, rank minimization problems.

2010 Mathematics Subject Classification. 15A, 46N10, 65K10, 90C.

References

[1] U.Helmke and J.B.Moore, *Optimization and Dynamical Systems*, Spinger Verlag (1994).

[2] N.Higham, *Matrix nearness problems and applications*, In M.J.C Gover and S.Barnett, editors, *Applications of Matrix Theory*, Oxford University Press (1989), 1-27.

[3] J.-B.Hiriart-Urruty and J.Malick, *A fresh variational analysis look at the world of the positive semidefinite matrices*, J. of Optimization Theory and Applications, Vol 153(3) (2012), 551–577 (Survey paper).

[4] J.-B.Hiriart-Urruty and H.Y.Le, *A variational look at the rank function*, to appear in TOP (Journal of the Spanish Society of Statistics and Operations Research), July 2013 (Survey paper).

[5] J.-J.Moreau, *Fonctions convexes duales et points proximaux dans un espace hilbertien.* (French) C. R. Acad. Sci. Paris 255 (1962) 2897–2899.

[6] J.-J.Moreau, *Propriétés des applications"prox"*. (French) C. R. Acad. Sci. Paris 256 (1963) 1069–1071.

[1]Institute of Mathematics
Paul Sabatier University, Toulouse (France)
jbhu@math.univ-toulouse.fr,http://www.math.univ-toulouse.fr/~jbhu/

[2]Institute of Mathematics
Paul Sabatier University, Toulouse (France)
hyle@math.univ-toulouse.fr

[7] R.T.Rockafellar and R.J-B.Wets, *Variational analysis*, Springer (1998).

[8] G.W.Stewart, *Matrix algorithms. Vol. I. Basic decompositions.* Society for Industrial and Applied Mathematics, Philadelphia, PA (1998).

Numerical Methods for Multi-Objective Optimal Control Problems

List of authors:
Yalcin Kaya [1]
<u>Helmut Maurer</u> [2]

We propose numerical methods for solving nonconvex multi–objective optimal control problems with control and state constraints. We employ a scalarization technique which reduces the problem to a single-objective optimal control problem. In contrast to a standard weighted-sum scalarization [1], we use a weighted Tschebychev scalarization that is particularly suited for nonconvex problems [2]. The weighted Tschebychev scalarization is surjective from the space of weights to the Pareto set (front). Solutions (obtained via discretization) of a sequence of scalarized problems yield an approximation of the Pareto front. The numerical method is illustrated on two numerically challenging problems involving tumor anti-angiogenesis [4] and a fedbatch bioreactor [5]. The control problems exhibit bang-bang and singular controls as well as boundary controls for the state constraints.

References

[1] H. Bonnel and Y. C. Kaya, Optimization over th efficient set of convex multi-objective optimal control problems. *J. Optimization Theory and Applications* **147**, 93–112 (2010).

[2] Y. Kaya and H. Maurer, A numerical method for generating the Pareto front of nonconvex multi-objective optimal control problems. submitted, 2013.

[3] J. Jahn, *Vector Optimization: Theory, Applications, and Extensions*, Springer, Berlin, 2010.

[4] U. Ledzewiz, H. Maurer and H. Schättler, Optimal and suboptimal protocols for a mathematical model for tumor anti-angiogenesis in combination with chemotherapy. *Mathematical Bioscieneces and Engineering* **8**, 307–323, 2011.

[5] F. Logist, B. Houska, M. Diehl and J. van Impe, Fast Pareto set generation for nonlinear optimal control problems with multiple objectives, *Struct. Multidisciplinary Optimization* **42**, 591–603, 2010.

[1] University of South Australia, School of Information Technology and Mathematical Sciences, Mawson Lakes, Australia
[2] University of Muenster, Institute of Computational and Applied Mathematics, Münster, Germany

Error Bounds and Hölder Metric Subregularity

List of authors:
A. Kruger [1]

Hölder metric subregularity/calmness of set-valued mappings can be treated in the framework of the theory of error bounds of real-valued functions. For this purpose, the machinery of error bounds needs to be extended to functions defined on the product of (metric or normed) spaces. Suppose X and Y are metric spaces, $(\bar{x},\bar{y}) \in X \times Y$, $f : X \times Y \to \mathbb{R} \cup \{+\infty\}$, $f(\bar{x},\bar{y}) = 0$, $f(x,y) > 0$ if $y \neq \bar{y}$, and $\liminf\limits_{f(x,y)\downarrow 0} \dfrac{f(x,y)}{d(y,\bar{y})} > 0$. Denote $S_f := \{x \in X \mid f(x,\bar{y}) \le 0\}$. We say that f has a local *error bound* with respect to x at $(\bar{x},\bar{y})$ if there exists a $\tau > 0$ such that

$$\tau d(x, S_f) \le f_+(x,y) \quad \text{for all } x \text{ near } \bar{x} \text{ and } y \in Y,$$

or, in other words,

$$\mathrm{Er}\, f(\bar{x},\bar{y}) := \liminf_{\substack{x\to\bar{x}\\ f(x,y)>0}} \frac{f(x,y)}{d(x,S_f)} > 0.$$

If $f(x,y) < \infty$ and $\rho > 0$, the *nonlocal ρ-slope* of f at (x,y) is defined as

$$|\nabla f|^{\diamond}_{\rho}(x,y) := \sup_{(u,v)\neq(x,y)} \frac{[f(x,y) - f_+(u,v)]_+}{d_\rho((x,y),(u,v))}, \tag{1}$$

where $d_\rho((x,y),(u,v)) := \max\{d(x,u), \rho d(y,v)\}$. Using (1), one can define the *uniform strict slope* of f at $(\bar{x},\bar{y})$:

$$\overline{|\nabla f|}^{\diamond}(\bar{x},\bar{y}) := \lim_{\rho\downarrow 0} \inf_{\substack{d(x,\bar{x})<\rho,\, f(x,y)<\rho\\ f(x,y)<d(x,S_f)^{1-\rho}}} |\nabla f|^{\diamond}_{\rho}(x,y). \tag{2}$$

Theorem 1 *(i)* $\mathrm{Er}\, f(\bar{x},\bar{y}) \le \overline{|\nabla f|}^{\diamond}(\bar{x},\bar{y})$*;*

(ii) if X and Y are complete and f_+ is lower semicontinuous near $(\bar{x},\bar{y})$, then $\mathrm{Er}\, f(\bar{x},\bar{y}) = \overline{|\nabla f|}^{\diamond}(\bar{x},\bar{y})$.

Let $q \in (0,1]$. A multifunction $F : X \to 2^Y$ between metric spaces is called *Hölder metrically subregular* (of order q) at $(\bar{x},\bar{y}) \in \mathrm{gph}\, F$ if there exists a $\tau > 0$ such that

$$\tau d(x, F^{-1}(\bar{y})) \le (d(\bar{y}, F(x)))^q \quad \text{for all } x \text{ near } \bar{x},$$

[1] Centre for Informatics and Applied Optimization, School of Science, Information Technology and Engineering, University of Ballarat, Australia

or, in other words,

$$^s r_q[F](\bar{x},\bar{y}) := \liminf_{\substack{x\to\bar{x}\\ x\notin F^{-1}(\bar{y})}} \frac{(d(\bar{y},F(x)))^q}{d(x,F^{-1}(\bar{y}))} > 0.$$

This property can be treated as a special case of the local error bound property for the function $f : X \times Y \to \mathbb{R} \cup \{+\infty\}$ defined by

$$f(x,y) = \begin{cases} (d(y,\bar{y}))^q & \text{if } (x,y) \in \operatorname{gph} F,\\ +\infty & \text{otherwise,} \end{cases}$$

as $^s r_q[F](\bar{x},\bar{y}) = \operatorname{Er} f(\bar{x},\bar{y})$. Slopes (1) and (2) take the following form:

$$|\nabla F|^{\diamond}_{q,\rho}(x,y) := \sup_{\substack{(u,v)\neq(x,y)\\ (u,v)\in\operatorname{gph} F}} \frac{[(d(y,\bar{y}))^q - (d(v,\bar{y}))^q]_+}{d_\rho((u,v),(x,y))} \quad (\rho > 0),$$

$$\overline{|\nabla F|}{}^{\diamond}_{q}(\bar{x},\bar{y}) := \lim_{\rho\downarrow 0} \inf_{\substack{d(x,\bar{x})<\rho,\, d(y,\bar{y})<\rho\\ (d(y,\bar{y}))^q<d(x,F^{-1}(\bar{y}))^{1-\rho}\\ (x,y)\in\operatorname{gph} F}} |\nabla F|^{\diamond}_{q,\rho}(x,y). \tag{3}$$

The above constants are called, respectively, the *nonlocal* (q,ρ)*-slope* of F at $(x,y) \in \operatorname{gph} F$ and the *uniform strict* q*-slope* of F at $(\bar{x},\bar{y}) \in \operatorname{gph} F$.

The next theorem is a consequence of Theorem 1.

Theorem 2 *(i)* $^s r_q[F](\bar{x},\bar{y}) \le \overline{|\nabla F|}{}^{\diamond}_{q}(\bar{x},\bar{y})$;

(ii) if X and Y are complete and $\operatorname{gph} F$ *is locally closed near* $(\bar{x},\bar{y})$*, then* $^s r_q[F](\bar{x},\bar{y}) \ge \overline{|\nabla F|}{}^{\diamond +}_{q}(\bar{x},\bar{y})$.

Several kinds of local primal space and coderivative sufficient conditions of Hölder metric subregularity can be deduced from Theorem 2 by providing lower estimates for the uniform strict q-slope (3) in terms of appropriate local slopes.

The research was partially supported by the Australian Research Council, project DP110102011.

References

[1] Fabian, M.; Henrion, R.; Kruger, A. Y. and Outrata, J. Error bounds: necessary and sufficient conditions. *Set-Valued Var. Anal.*, 18:121–149, 2010.

[2] Bednarczuk, E. M. and Kruger, A. Y. Error bounds for vector-valued functions: necessary and sufficient conditions. *Nonlinear Anal.* 75:1124–1140, 2012.

[3] Ngai, H. V.; Kruger, A. Y. and Théra, M. Slopes of multifunctions and extensions of metric regularity. *Vietnam J. Math.*, 40:355–369, 2012.

Maximising Sensitivity of Electrical Impedance of a Piezoelectic Ceramic to Material Parameters using Modified Electrode Configuration

List of authors:

K. Kulshreshtha [1]

J. Rautenberg [2] C. Unverzagt [3]

Piezoceramics are often used in ultrasonic measurement devices (e.g. for flow, material concentration or level measurement as well as for non-destructive testing). To increase the robustness and functionality of the ultrasonic sensors, their development is often supported by computer simulations. These finite element based simulations are highly sensitive to simulation parameters, especially the material dataset of the modeled piezoceramic [**?**, **?**]. However, these constants are rarely well known as each produced charge of piezoceramics has its own parameters. These parameters must be identified to reach optimal sensor design.

One cheap and non-intrusive method for parameter estimation is using the measurement of the electrical impedance characteristics of the ceramic. In usual electrode topologies, however, the impedance characteristics show little or no sensitivity to changes in certain critical parameters [**?**]. By applying a non-uniform electric potential using ring shaped electrodes, an increase in the sensitivity was observed and the corresponding results have been presented in [**?**].

In order to apply well known parameter estimation techniques, e.g. as discussed in [**?**] one requires the sensitivity to be as high as possible. The optimisation problem is, in this case, to maximize the sensitivity by adapting the electrode configuration, before the inverse problem can be solved in the best setup. This optimisation problem has the sensitivity, which is a derivative of the impedance, as its objective function and thus higher derivatives are required if one uses the usual methods for nonlinear programming. Computation of higher derivatives is not always economical and derivative free optimisation methods are of significant interest in such cases. This talk will present the results for such an optimisation problem, where the sensitivity is maximized in preparation for the solution of an inverse problem.

References

[1] M.Kaltenbacher: Simulationsbasierte Entwicklung von Sensoren. In: Technisches Messen 79, pp. 30-36, 2012.

[2] B.Henning, J.Rautenberg, C.Unverzagt, A.Schrder, S.Olfert: Computer-assisted design of transducers for ultrasonic sensor systems. In: Meas. Sci. Technol., Vol. 20 (124012), Issue 12, 2009.

[1] University of Paderborn
[2] University of Paderborn
[3] University of Paderborn

[3] D.Kybartas, A.Lukosevicius: Determination of piezoceramics parameters by the use of mode interaction and fitting of impedance characteristics. In: Ultragarsas 45 (4), pp. 2228, 2009.

[4] Carsten Unverzagt, Jens Rautenberg, Bernd Henning, and Kshitij Kulshreshtha. Modified electrode shape for the improved determination of piezoelectric material parameters. In Gan Woon Siong, Lim Siak Piang, and Khoo Boo Cheong, editors, Proceedings of the 2013 International Congress on Ultrasonics. 2013.

[5] B.Kaltenbacher, A.Neubauer, and O.Scherzer: Iterative Regularization Methods for Nonlinear Problems. de Gruyter, Berlin, New York, 2008.

Optimal Discrete-Time Finite-Dimensional Inertial Filters

List of authors:

Andrzej Latocha [1]

This talk will address optimal estimation of the expected value. This paper discusses the problem of estimation relating the expected value of a signal that is subject to stochastic noises. An innovative solution in the proposed algorithm based a non-standard approach. In proposed non-standard approach the mathematical model isn't known and unknown is the distribution of noise. This paper aims at devising a transparent and effective algorithm to estimate the expected value based on discrete measurement data, i.e. an algorithm that is well established mathematically and numerically for practical applications in standard SRT (SoftReal-Time) computational systems. Stochastic noises constitute an inseparable element the process in automation systems. Noises determine proper operation of control systems and their accuracy. In classic systems, noise filtration consists in application of observers or the Kalman filter [1][2]. The necessity to know a mathematical model of the dynamic system that is the source of the degraded signal, and the necessity to solve complex differential and integral equations, make such solutions disadvantageous. Other solutions to the filtration problems are based on knowing parameters of distribution of discrete measurement samples [1]. In dynamic systems, a mathematical model of such a system is often not determined. Distribution of the sampled data is arranged outside an undetermined curve. An application of classic filtration methods is difficult to be implemented. By using the methods of linear regression, it is possible to devise estimation algorithms for the expected value of a process signal that is subject to stochastic noises. The selection of linear regression is justified by the necessity to identify signals that do not belong to elementary functions as well as the criterion of feasibility in computer-based control systems. In order to satisfy the mentioned requirements the algorithm should be characterised by good mathematical and numerical conditions in the current computational systems [3]. In order to carry out the task the method of the least squares (LS) has been applied. The final solution was obtained by the recursive of narrow noise variance estimation of the distribution. The advantage of LS is fast convergence of the algorithm comparable with the dynamics of convergence of classical observers and solutions finite-dimensional. Acquisition of an optimal parametric solution and fast convergence of the algorithm weigh in favour of the method of least squares. The suggested algorithm can be applied to control, diagnostic, and predictive systems. The algorithm does not necessitate determining a mathematical model of the dynamic system compared with[2] and distribution parameters of measurement samples. Advantages of the algorithm include transparency, effectiveness in standard computational systems, a slight load of the estimator for degraded Gauss distributions, immunity to noises. Increased accuracy, precision in control systems opens new horizons, reveals problems which has not been seen, allows the application of algorithms which so far could not be used.

[1]Department of Automatics and Biomedical Engineering, AGH University of Science and Technology, Krakow, Poland

References

[1] Theodore Alexandrov, Bianconcini Silvia, Bee Dagum Estela, Maass Peter, McElroy Tucker.: A Review of Some Modern Approaches to the Problem of Trend Extraction. Washington 2008. Center of Industrial Mathematics, University of Bremen; Department of Statistics, University of Bologna.

[2] Elliott Robert.J., Krishnamurthy Vikram.: Exact Finite-Dimensional Filters For Maximum Likelihood Parameter Estimation Of Continuous-Time Linear Gaussian Systems. Siam J. Control Optim. Society for Industrial and Applied Mathematics 1997. November Vol. 35, No. 6, pp. 1908-1923

[3] Latocha Andrzej.: The use of Data Exchange Sstandard OPC DA, OPC DX Layered Control Systems to Implement Non-Standard Task Automation through Integration PLC with Advanced Software. Kielce 2011. XVII National Conference on Automation - KKA2011, Monographs Committee for Automation and Robotics PAN.

[4] Soderstrom Torsten, Stoica Petre.: System Identification. Warszawa 1997. PWN translated by Banasiak J.

On the robustification of optimum experimental design problems

List of authors:

M. S. Mommer [1]

Optimum experimental design (OED) is the problem of finding setups for an experiment in such a way that the collected data allows for optimally accurate estimation of the parameters of interest - taking into account an experimental budget. In practice, the parameters are only approximately known as a matter of course, while at the same time, solving an OED problem is in a way equivalent to magnifying the dependence of the system response on these quantities. As a consequence, designs computed on the bais of a "good guess" of the parameters may underperform dramatically in practice, especially for problems involving nonlinear models.

In this talk, we introduce new robust formulations for optimum experimental design that work under significant uncertainty, and compare their performance with existing robustified OED methods. Our focus is on problem settings in which the model is described by differential equations of some type that are solved numerically. Our approach is based on a semi-infinite programming formulation in which we exploit additional problem structure, together with sparse grids, to ensure tractability. We also show how to construct a formulation of robust experimental design that, in contrast to many OED formulation (even non-robust ones) requires no derivatives in its formulation, and as a consequence can be used with existing simulation software that is not able to compute sensitivities of order more than one.

The talk includes numerical experiments to illustrate and compare the effectiveness of the approaches.

[1]Simulation and Optimization group, Interdisciplinary Center for Scientific Computing (IWR), University of Heidelberg, Im Neuenheimer Feld 368, 69120 Heidelberg. `mommer@uni-heidelberg.de`

Simultaneous Size, Shape and Topology Optimization in Parallel Numerical Environment.

List of authors:

M. Nowak [1]

H. Hausa [2]

There are many examples of using optimization techniques to design the structural elements, but simultaneous size, shape and topology optimization is rather rare. In the resent years, especially the topology optimization method has been introduced to the designing processes. A good industrial example is here the structure of the Airbus A380 wing. The structural elements of the wing were designed in two designing steps [1]. First, the optimal material distribution was defined using the topology optimization. Then, after extraction of geometry from the topology optimization results, the model for size and shape optimizations was derived. Splitting the topology and then size and shape optimizations is necessary, due to completely different optimization methods used in each case. In this talk the structural optimization method in parallel numerical environment with some unique properties will be presented. The optimization system is based on the discussion of design with optimal stiffness [2], which leads to the conclusion, that for the stiffest design strain energy density (SED) along the shape to be designed must be constant. To achieve postulated constant SED value on the structural surface the biomimetic optimization system was designed. The numerical simulation system presented in the talk is based on the algorithm of bone remodeling stimulated by mechanical loading [3]. Adaptation to mechanical stimulation, and thereby equalization of the SED on the structural surface, results in altering the structural surface position in virtual space. This special kind of structural optimization has some specific features that distinguish it from the widely used methods such as SIMP. These features, which provide new possibilities in the area of structural optimization, like:

- the domain independence,

- functional configurations during the process of optimization,

- possibility to solve the multiple load problems,

allow to comprise optimizations of size, shape, and topology with no need to define parameters. The presented method is able to produce results similar to the standard method of topology optimization and can be useful in mechanical design, especially when functional structures are needed during the optimization process. Due to parallelisation of both the structural analysis of strain energy density distribution and volume mesh generation, the presented method can be useful in real industrial problems [4]. The SED computations are carried out in a parallel environment, which is a condition to solve bigger problems and the same question concerns mesh generation.

[1] Poznan University of Technology, Division of Virtual Engineering, ul. Piotrowo 3, 60-965 Poznan, Poland, e-mail: Michal.Nowak@put.poznan.pl

[2] Poznan University of Technology, Division of Virtual Engineering

The concept of Finite Element mesh parallel generation as well as Finite Element Analysis in a parallel environment will be briefly presented - to illustrate the usefulness of the method for real-world problems only.

This work was supported by the Polish National Science Centre under the grant - decision no. DEC-2011/01/B/ST8/06925.

References

[1] Krog L. et al., Topology optimization of aircraft wing box ribs *AIAA-Paper*, 2004-4481, 2004.

[2] Dzieniszewski W., Optymalizacja kształtów konstrukcji w założeniach teorii sprężystości. *Zeszyt IPPT PAN, Optymalizacja wytrzymałościowa konstrukcji*, Ossolineum: 114–137, 1983.

[3] Nowak M., A generic 3-dimensional system to mimic trabecular bone surface adaptation. *Computer Methods in Biomechanics and Biomechanical Engineering*, 9(5):313–317, 2006.

[4] Nowak M., From the Idea of Bone Remodelling Simulation to Parallel Structural Optimization. *Numerical Methods for Differential Equations, Optimization, and Technological Problems*, 335–344, 2013.

On a Global Search in Operations Research Problems with a Bilinear Structure

List of authors:
Andrei Orlov [1]

As known development of new efficient methods for the solving of nonconvex problems is the urgent problem of contemporary Operations Research. In this work we deal with the special class of nonconvex problems — the problems with a bilinear structure. The class includes bimatrix games and games with bilinear payoff functions, bilinear programming problems, problems of bilinear separability, bilevel problems etc. (see, for example, [1, 2, 3, 4]). Note, with the help of such problems the transportation and railway problems can be modeled.

For problems with the bilinear structure the new approach of global search is elaborated. The approach is based on two principal features of bilinear functions: any bilinear function is affine in each of its variables when the other variable is fixed; and it is represented as the difference of two convex functions (so, a bilinear function is d.c. function). Also the approach takes into account a possibility of equivalent representation of problems under scrutiny as nonconvex optimization problems. These nonconvex problems are solved by using the Global Search Theory proposed by A.S. Strekalovskiy [5]. Global Search Theory in problems with bilinear structure consists of two basic stages: 1) a special local search methods (LSM), which takes into account the first feature of bilinear functions; 2) the procedures, based on Global Optimality Conditions in d.c. optimization problems, which allow to improve the point provided by LSM.

So, in contrast to the widely distributed approaches to nonconvex problems such as branch & bound methods, cuts methods, outside and inside approximations methods, vertex enumeration, simulated annealing methods, genetic algorithms, ant colony algorithms and so on, our approach allow to use contemporary convex optimization methods within Local and Global Search Procedures. Also with the help of our approach we can solve nonconvex optimization problems of high dimension (up to hundreds variables) [1, 2, 3, 4].

Therefore the approach allows building efficient methods for finding global solutions in problems with bilinear structure. Computational testing of the elaborated methods has shown the efficiency of the approach. This work is carried out under financial support of RFBR (projects no. 13-01-92201_Mong_a, 12-07-33045-mol_a_ved, 12-07-13116-ofi_m_RZD).

References

[1] Orlov, A.V., Strekalovsky, A.S., Numerical Search for Equilibria in Bimatrix Games, *Computational Mathematics and Mathematical Physics*, 45(6):947–960, 2005.

[2] Orlov, A.V., Numerical solution of bilinear programming problems, *Computational Mathematics and Mathematical Physics*, 48(2):225–241, 2008.

[1] Institute for System Dynamics and Control Theory of SB RAS, Lermontov St., 134, Irkutsk, 664033, Russia

[3] Gruzdeva, T.V., Petrova, E.G., Numerical solution of a linear bilevel problem, *Computational Mathematics and Mathematical Physics*, 50(6):1631–1641, 2010.

[4] Orlov, A.V., Strekalovsky, A.S., Malyshev, A.V., On computational search for optimistic solutions in bilevel problems, *Journal of Global Optimization*, 48(1): 159–172, 2010.

[5] Strekalovsky, A.S., Elements of Nonconvex Optimization, *Nauka*, Novosibirsk, 2003. (in Russian)

Infinite Horizon Optimal Control Problems with Budget Constraints

List of authors:
Sabine Pickenhain [1]
Valeriya Lykina [2]

We consider a class of infinite horizon optimal control problems in Lagrange form involving the Lebesgue integral in the objective and isoperimeric constraints.

This special class of problems arises in the theory of economic growth and in processes where the time T is an exponentially distributed random variable.

The problem is formulated as optimization problem in Hilbert Spaces. It reads as follows: Minimize the functional

$$J(x,u) = \int_0^\infty r(t,x(t),u(t))\nu(t)dt$$

subject to all pairs $(x,u) \in W_2^{1,n}(\mathbb{R}^+,\nu) \times L_2^r(\mathbb{R}^+,\nu)$, satisfying
state equations

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)\,,\ x(0) = x_0,$$

control restrictions

$$u(t) \in U, \quad U \in Comp\,(R^r) \setminus \{\,\emptyset\,\},$$

and an isomperimetric constraint

$$\int_0^\infty c^T(t)x(t)\nu(t)dt = D.$$

The remarkable on this statement is the choice of Weighted Sobolev- and Weighted Lebesgue spaces as state and control spaces respectively. The function ν is a density function. These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a Hilbert space.

For the class of problems proposed, we prove a Pontryagin type Maximum Principle and give applications.

[1]BTU Cottbus, Germany
[2]BTU Cottbus, Germany

Identification, Modeling and Optimization of Active Magnetic Levitation Electromagnet

List of authors:
Adam Piłat [1]

This work presents the investigation of a cylindrical electromagnet designed for the Active Magnetic Levitation System. The aim of the experimental stage is to obtain the magnetic flux density close to the electromagnet core and the electromagnet coil current. The experimental investigation in the open automatic control chain was supported by the Programmable Analog Controller (PAC) [1]. It is well known that the hysteresis can be cancelled by a current feedback. One can find such a solution implemented in the hardware layer of PAC [2]. The Active Magnetic Levitation electromagnet was modeled with the support of the Finite Element Method. Due to the cylindrical construction of the electromagnet the model was built in the axis symmetry mode to simplify calculations. Therefore, the manufactured electromagnet cross-section was reflected in the model geometry. Additionally, the electrical circuit representing the voltage driven electromagnet coil was modeled. The calculated coil current was used to drive the electromagnet and to expand the levitation model [3]. The presence of hysteresis effect was considered. The magnetic field problem together with the coil excitation by the voltage control signal was solved in the time domain. On the basis of the designed model, the optimization problem devoted to the electromagnetic force maximization under geometry and control constraints was formulated. Finally, the possibility of controller synthesis on the basis of such a model was analyzed.

References

[1] Piłat, A.: The programmable analog controller : static and dynamic configuration, as exemplified for active magnetic levitation. *Przegląd Elektrotechniczny*, Vol. 88, no 4b, 2012, 282-287

[2] Piłat A., Coil Current Proportional Feedback Embedded into Programmable Analog Controller. MMAR 2013, Międzyzdroje

[3] Piłat A., Modelling, Investigation, Simulation, and PID Current Control of Active Magnetic Levitation FEM Model. MMAR 2013, Międzyzdroje

[4] Jiles D.C., Atherton D.L., Ferromagnetic Hysteresis. *IEEE Transactions on Magnetics*, March 19, 1983

[1]Department of Automatics and Biomedical Engineering, AGH University of Science and Technology, Krakow, Poland

New Methods for Solving Nonconvex Optimal Control Problems

List of authors:
Alexander Strekalovskiy [1]

As known, the most of real-life optimization problems turn out to be both nonconvex and dynamic, what provides a huge of difficulties and singularities in studying and moreover in a numeric search for a global solution to the problems.

We consider optimal control (OC) problems with quadratic functionals defined by matrices which are indefinite, otherwise the problems with integral and terminal (d.c.) functionals, representable as a difference of two convex functionals (Bolza problems). For this class of optimal control problems we propose, first, special local search methods, consisting in consecutive solving the linearized (w.r.t. the basic nonconvexities at a current iteration) problems, and a study of its convergence [6]. Second, for the OC problems under scrutiny Global Optimality Conditions (GOC) developed, from which in particular the Pontryagin maximum principle follows [4, 5, 9]. On the base of these GOC a family of Global Search Methods (GSM) have been developed and its convergence was investigated [7, 8].

Besides a family of Local Search methods special for each kind of nonconvexity was proposed and substantiated, and after that incorporated into Global Search Procedures [3].

Further the number of special nonconvex OC test problems has been generated by the procedure the idea of which belong to L.N.Vicente and P.H.Calamai [1, 2].

On this large field of benchmarks problems some of that are of rather high dimension (20 in state and 20 in control) it was conducted a large number of computational experiments which witnessed on the attractive abilities and the promising effectiveness of the developed approach.

References

[1] Vicente, L.N., Calamai, P.H., and Judice, J.J., Generation of Disjointly Constrained Bilinear Programming Test Problems, *Comput. Optimizat. and Applic.*, 1(3):299–306, 1992.

[2] Calamai, P.H. and Vicente, L.N. Generating Quadratic Bilevel Programming Test Problems, *ACM Transaction in Mathematical Software*, 20:103–119, 1994.

[3] Strekalovsky, A.S., Elements of Nonconvex Optimization, *Nauka*, Novosibirsk, 2003. (in Russian)

[4] Strekalovsky, A.S., Global Optimality Conditions for Optimal Control Problems with Functions of A.D.Alexandrov, *Journal of Optimization Theory and Applications*, 2013. (submitted)

[1] Institute for System Dynamics and Control Theory of SB RAS, Lermontov St., 134, Irkutsk, 664033, Russia

[5] Strekalovsky, A.S., Maximizing a State Convex Lagrange Functional in Optimal Control, *Automation and Remote Control*, 73(6):949–961, 2012.

[6] Strekalovsky, A.S., Local Search for Nonconvex Optimal Control Problems of Bolza, *Numerical Methods and Programming*, 11:344–350, 2010.

[7] Strekalovsky, A.S. and Yanulevich, M.V., Global Search in the Optimal Control Problem with a Terminal Objective Functional Represented as the difference of two convex functions, *Comput. Math. Math. Phys.*, 48(7):1119–1132, 2008.

[8] Strekalovsky, A.S. and Sharankhaeva, E.V., Global search in a nonconvex optimal control problem, *Comput. Math. Math. Phys.*, 45(10):1719–1734, 2005.

[9] Strekalovsky, A.S., On global maximum of a convex terminal functional in optimal control problems, *Journal of Global Optimization*, 7:75–91, 1995.

High order optimality conditions for p-regular inequality constrained optimization problem

List of authors:

E. Szczepanik [1]

A. Prusińska [2]

A. Tret'yakov [3]

Consider the following optimization problem

$$\min \varphi(x) \tag{1}$$

subject to

$$g_i(x) \le 0, i = 1, ..., m, \tag{2}$$

where $x \in \mathbb{R}^n$, $\varphi : \mathbb{R}^n \to \mathbb{R}$, $\varphi \in \mathcal{C}^2(\mathbb{R}^n)$, $g_i : \mathbb{R}^n \to \mathbb{R}$ and $g_i \in \mathcal{C}^{p+1}(\mathbb{R}^n), p \in \mathbb{N}$.

Let x^* be the solution of (1)–(2) and $X = \{x \in \mathbb{R}^n | g_i(x) \le 0,\ i = 1, \dots, m\}$.
Assume that the active constraints $g_i(x^*)$ that forming the index set
$I(x^*) = \{i \in \{1, \dots, m\} \mid g_i(x^*) = 0\} \stackrel{\triangle}{=} \{1, \dots, l\}$, $l \le m$, are irregular (nonregular, singular, degenerate) at x^*, i.e. $\{g_i'(x^*)\}_{i \in I(x^*)}$ are linearly dependent.

Let $g = (g_1, \dots, g_{r_1}, g_{r_1+1}, \dots, g_{r_2}, \dots, g_{r_{p-1}+1}, \dots, g_{r_p})^T$, $r_p = l$, and

$$\begin{array}{lcll} g_i'(x^*) & = & 0, & i = r_1 + 1, \dots, l, \\ g''_i(x^*) & = & 0, & i = r_2 + 1, \dots, l, \\ \dots\dots\dots & \dots & \dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ g_i^{(p-1)}(x^*) & = & 0, & i = r_{p-1} + 1, \dots, r_p = l. \end{array} \tag{3}$$

The operators of higher derivatives $\left\{g_i^{(k)}(x^*)\right\}_{i=r_{k-1}+1,\dots,r_k}$ are linearly independent, $k = 1, \dots, p$, $r_0 \stackrel{\triangle}{=} 0$. The constraints can be written in the form (3) by applying linear transformations of $g(x)$.

We give a description of the critical cone for the problem (1)–(2) and the outer boundary of this cone. Moreover, we formulate Karash-Kuhn-Tucker type optimality conditions with the help of the p-factor operator, which is the main construction of the p-regularity theory.

[1] Siedlce University, Siedlce, Poland; email: ewa.szczepanik@ii.uph.edu.pl
[2] Siedlce University, Siedlce, Poland; email: aprus@uph.edu.pl
[3] Siedlce University, Siedlce; System Research Institute, Polish Academy of Sciences, Warsaw, Poland; Dorodnitsyn Computing Center, Russian Academy of Sciences, Moscow, Russia; email: tret@uph.edu.pl

Penalty Aided Transitions to Optimal Control Structures in the MSE

List of authors:
Maciej Szymkat [1]
Adam Korytowski [2]

The method of Monotone Structural Evolution (MSE) belongs to a wide class of *gradient* computational methods of optimal control, known as *direct sequential*. It has several distinctive features. The number and, possibly, type of optimization variables are changed during the optimization in a systematic way. Periods of gradient optimization in a constant decision space are interrupted by structural changes, which alter the optimization space without affecting the current approximation of optimal control. The performance index monotonically decreases, and the stationary points of the algorithm have to satisfy the necessary optimality conditions of the maximum principle. Special rules prevent the algorithm from convergence to chattering modes. With some preparatory work provided (which consists in defining a sufficiently rich stock of *MP-consistent* control procedures), the MSE gives a good chance to reveal an optimal control structure in an automatic way. For more information, see [1, 2, 3].

However, using the whole apparatus of the MSE from the very start is not always advantageous. As long as the current solution is far from optimal, it is usually more convenient to confine the stock of control procedures to *approximative* ones and freeze the optimization space (the control structure). This reduces the MSE to the well known direct sequential methods. The full power of the MSE should be switched on *at the right moment*. The criteria for choosing such a moment are one of the subjects of this work.

It may happen, especially if the switching moment is chosen too late, that the gradient forces become too weak for the MP-consistent control procedures to expand and to replace the approximative ones in order to create the optimal control structure. On the other hand, if this is done too early, the current estimates of optimal adjoints may be insufficiently accurate for a proper initialization of parameters of some MP-consistent procedures. A tool that can be helpful in such a situation was proposed in [3], where it was called the *transition method*.

This work will be devoted to a systematic study of this problem. New transition methods will be described based on a penalty approach, with *flat* (as in [3]) and *spike* generations of appropriate consistent procedures. The penalty will be ascribed to those control arcs which are represented by control procedures which certainly do not appear in an optimal control structure. As a result, a method is obtained which transforms a typical numerical approximation of optimal control into a representation with explicit optimal control structure.

[1]Department of Applied Computer Science, AGH University of Science and Technology, Krakow, Poland
[2]Department of Automatics and Biomedical Engineering, AGH University of Science and Technology, Krakow, Poland

References

[1] M. Szymkat, A. Korytowski: *Method of monotone structural evolution for control and state constrained optimal control problems*. European Control Conference ECC 2003, University of Cambridge, U.K., September 1-4, 2003

[2] A. Korytowski, M. Szymkat: Consistent Control Procedures in the Monotone Structural Evolution. Part 1: Theory. In: M. Diehl et al. (eds.), *Recent Advances in Optimization and its Applications in Engineering*, Springer-Verlag, Berlin Heidelberg 2010, 247-256.

[3] M. Szymkat, A.Korytowski: *Control preserving transition from approximative to consistent procedures in direct dynamic optimization*. 15th Austrian-French-German Conference on Optimization, September 19-23, 2011, Toulouse.

Stability, Controllability and Optimization of a Model of Treatment Response to Combined Anticancer Therapies.

List of authors:
A.Świerniak, [1]
J. Klamka [2]

In the paper, we consider a second-order finite-dimensional semilinear stationary dynamical system described by a set of two ordinary differential state equations. More precisely, we discuss the control properties of a model belonging to a class proposed by Hahnfeldt et al. in [1] to which one control variable in the case of antiangiogenic therapy or two control variables describing two treatment modalities (antiangiogenic and chemo- therapies) have been introduced. The Hahnfeldt et al. model [1] is based on the assumption that tumor growth with an incorporated vascularization mechanism can be described by a Gompertz-type or logistic-type equation with variable carrying capacity which defines the dynamics of the vascular network. The main idea of this class of models is to incorporate the spatial aspects of the diffusion of factors that stimulate and inhibit angiogenesis into a non-spatial two-compartmental model for cancer cells and vascular endothelial cells. More precisely we study a model which is a modification of original Hahnfeldt et al model proposed in [2] . In the case of stability we follow the line of reasoning proposed in [2] . First we find a non-trivial equillibrium for control free system and prove its local stability using linearization analysis. Then an energy type Lyapunov function is used to prove global stability of the system. The next steps include asymptotic analysis of the model behavior in the case of constant and periodic treatments administered in the infinite control intervals.

The practical question which arises in this case is how these results could be applied in the case of realistic finite treatment horizons. All the considerations related to finite time treatment are conditioned on the concept of controllability of the dynamical systems discussed which, to our knowledge, has not been analyzed by other authors except in our previous paper [4]. We prove that the model with two controls is locally constrained controllable. The results are based on theorems proved in [3]. The idea of the theorems is that under suitable assumptions the constrained global controllability of a linear associated approximated dynamical system implies constrained local relative controllability near the origin of the original semilinear second-order dynamical system.

Using Potryagin maximum principle we find treatment protocols which satisfy necessary conditions of optimality. The control strategy is found to be a bang-bang one that means that only switchings between full dose and non dose administrations are optimal. Singular arcs are found to be non optimal using generalized Clebsch-Legendre and Goh conditions. Our result is in this case different than the one obtained by Ledzewicz and Schattler [5] who proved that for the original Hahnfeldt model optimal trajectory should contain singular arcs. The theoretical results are illustrated by simulation experiments for biologically justified sets of parameters.

[1] Department of Automatic Control, Silesian University of Technology, Gliwice, Poland
[2] Department of Automatic Control, Silesian University of Technology, Gliwice, Poland

References

[1] P. Hahnfeldt, D. Panigraphy, J. Folkman and L. Hlatky. Tumor development under angiogenic signaling: A dynamic theory of tumor growth, treatment response and postvascular dormacy, *Cancer Research*, vol.59, 4770-4778, (1999)

[2] A. D'Onofrio and A. Gandolfi. Tumour eradication by antiangiogenic therapy analysis and extensions of the model by Hahnfeldt et al, *Math. Biosci.* 191, 159-184, (2004)

[3] J. Klamka. Constrained controllability of nonlinear systems. *J. Math. Anal. Appl.*, vol. 201, 365-374, (1996)

[4] A. Swierniak and J. Klamka. Control properties of models of antiangiogenic therapy, *Advances in Automatics and Robotics* (K. Malinowski and R. Dindorf Eds.), Monograph of Committee of Automatics and Robotics PAS, vol.16, Kielce, pt.2, 300-312, (2011)

[5] U. Ledzewicz and H. Schaettler Anti-angiogenic therapy in cancer treatment as an optimal control problem, *SIAM J. Contr. Optim*, 46, 1052-1079, (2007)

Pontryagins Maximum Principle for Infinite Horizon Optimal Control Problems with a nonlinear dynamical system

List of authors:

N. Tauchnitz [1]

In this talk we present the Pontryagins Maximum Principle for the infinite horizon optimal control problem

$$J\big(x(\cdot),u(\cdot)\big) = \text{(L-)} \int_0^\infty f\big(t,x(t),u(t)\big)\,dt \to \inf$$

subject to the state equation

$$\dot{x}(t) = \varphi\big(t,x(t),u(t)\big), \quad x(0) = x_0,$$

and control restrictions

$$u(t) \in U \subseteq \mathbb{R}^m, \quad U \neq \emptyset.$$

The integral in the functional J denotes the Lebesgue integral.

We consider $\varphi\big(t,x(t),u(t)\big) = \sqrt{x(t)} - x(t)$, $x(0) = 2$, and remark that the solution $x(t) = \big[1 + (\sqrt{2}-1)e^{-\frac{1}{2}t}\big]^2$ doesn't belong to the space $W^{1,2}(\mathbb{R}_+,\mathbb{R})$. Therefore we investigate the optimal control problem on the Weighted Sobolev space $W^{1,2}_\nu(\mathbb{R}_+,\mathbb{R}^n)$,

$$\begin{aligned} W^{1,2}_\nu(\mathbb{R}_+,\mathbb{R}^n) := \{x(\cdot) \quad | \quad & x(\cdot) : \mathbb{R}_+ \to \mathbb{R}^n \text{ measurable}, \\ & \int_0^\infty \big(\|x(t)\|^2 + \|\dot{x}(t)\|\big)\nu(t)\,dt < \infty\}, \quad \nu(t) = e^{-at}\ (a>0), \end{aligned}$$

as the state space.

The result is the statement of Pontryagins maximum principle in the normal form with an adjoint variable which belongs to the Weighted space $W^{1,2}_{\nu^{-1}}(\mathbb{R}_+,\mathbb{R}^n)$.

[1] Department of Mathematics, BTU Cottbus
PF 10 13 44, 03013 Cottbus, Germany
tauchnitz@math.tu-cottbus.de

Nonparametric Instrumental Regression with Non-Convex Constraints

List of authors:

A. Vanhems [1]

M. Grasmair [2]

O. Scherzer [3]

This paper considers the nonparametric regression model with an additive error that is dependent on the explanatory variables. As is common in empirical studies in epidemiology and economics, it also supposes that valid instrumental variables are observed. A classical example in microeconomics considers the consumer demand function as a function of the price of goods and the income, both variables often considered as endogenous. In this framework, the economic theory also imposes shape restrictions on the demand function, like integrability conditions. Motivated by this illustration in microeconomics, we study an estimator of a nonparametric constrained regression function using instrumental variables by means of Tikhonov regularization. We derive rates of convergence for the regularized model both in a deterministic and stochastic setting under the assumption that the true regression function satisfies a projected source condition including, because of the non-convexity of the imposed constraints, an additional smallness condition.

References

[1] R. A. Adams. *Sobolev Spaces.* Academic Press, New York, 1975.

[2] J.D. Angrist and A.B. Krueger. Instrumental variables and the search for identifications: from supply and demand to natural experiments. *J. Econ. Persp.*, 15(4):69–85, 2001.

[3] G. Aubert and L. Vese. A variational method in image recovery. *SIAM J. Numer. Anal.*, 34(5):1948–1979, 1997.

[4] N. Bissantz, T. Hohage, and A. Munk. Consistency and rates of convergence of nonlinear Tikhonov regularization with random noise. *Inverse Probl.*, 20(6):1773–1789, 2004.

[5] N. Bissantz, T. Hohage, A. Munk, and F. Ruymgaart. Convergence rates of general regularization methods for statistical inverse problems and applications. *SIAM J. Numer. Anal.*, 45(6):2610–2636 (electronic), 2007.

[6] M. Burger and S. Osher. Convergence rates of convex variational regularization. *Inverse Probl.*, 20(5):1411–1421, 2004.

[1] University of Toulouse, Toulouse Business School and Toulouse School of Economics, France
[2] Computational Science Center, University of Vienna, Austria
[3] Radon Institute of Computational and Applied Mathematics, Linz, Austria

[7] G. Chavent and K. Kunisch. Convergence of Tikhonov regularization for constrained ill-posed inverse problems. *Inverse Probl.*, 10:63–76, 1994.

[8] F. H. Clarke, Yu. S. Ledyaev, R. J. Stern, and P. R. Wolenski. *Nonsmooth analysis and control theory*, volume 178 of *Graduate Texts in Mathematics.* Springer-Verlag, New York, 1998.

[9] B. Eicke. Iteration methods for convexly constrained ill-posed problems in Hilbert space. *Numer. Funct. Anal. Optim.*, 13(5-6):413–429, 1992.

[10] J. Flemming and B. Hofmann. Convergence rates in constrained Tikhonov regularization: equivalence of projected source conditions and variational inequalities. *Inverse Probl.*, 27(8):085001, 11, 2011.

[11] M. Grasmair. Generalized Bregman distances and convergence rates for non-convex regularization methods. *Inverse Probl.*, 26(11):115014, 2010.

[12] P. Hall and J.L. Horowitz. Nonparametric methods for inference in the presence of instrumental variables. *Ann. Statist.*, 33(6):2904–2929, 2005.

[13] B. Hofmann, B. Kaltenbacher, C. Pöschl, and O. Scherzer. A convergence rates result for Tikhonov regularization in Banach spaces with non-smooth operators. *Inverse Probl.*, 23(3):987–1010, 2007.

[14] H.-G. Müller. Smooth Optimum Kernel Estimators Near Endpoints. *Biometrika*, 78:521-530, 1991.

[15] A. Neubauer. Finite-dimensional approximation of constrained Tikhonov-regularized solutions of ill-posed linear operator equations. *Math. Comp.*, 48(178):565–583, 1987.

[16] A. Neubauer and O. Scherzer. Finite-dimensional approximation of Tikhonov regularized solutions of nonlinear ill-posed problems. *Numer. Funct. Anal. Optim.*, 11(1-2):85–99, 1990.

[17] O. Scherzer, M. Grasmair, H. Grossauer, M. Haltmeier, and F. Lenzen. *Variational methods in imaging*, volume 167 of *Applied Mathematical Sciences.* Springer, New York, 2009.

[18] A. Vanhems. Nonparametric estimation of exact consumer surplus with endogeneity in price. *Econometrics Journal*, 13(3):80-98, 2010.

[19] L. Vese. A study in the BV space of a denoising-deblurring variational problem. *Appl. Math. Optim.*, 44(2):131–161, 2001.

[20] W. P. Ziemer. *Weakly Differentiable Functions. Sobolev Spaces and Functions of Bounded Variation*, volume 120 of *Graduate Texts in Mathematics.* Springer Verlag, Berlin etc., 1989.

$\mathcal{H}_2$-norm Model Reduction for Time-Variant Systems and its Application to PDE Constrained Optimization

List of authors:
G. Vossen [1]

Model reduction with respect to the $\mathcal{H}_2$-norm (or, for unstable systems, $\mathcal{H}_{2,\alpha}$-norm) provides a strong computational benefit for the simulation of linear time-invariant (LTI) dynamical systems [1, 2]. In particular, optimization problems for LTI systems can be solved very efficiently by using this technique. A typical example is given by PDE constrained optimization or optimal control problems [3] obtained by spatial semi-discretization using methods such as finite differences, finite elements or discontinous Galerkin. However, $\mathcal{H}_2$-norm model reduction is designed for time-invariant systems.

In this talk, extensions to linear time-variant (LTV) systems will be discussed. The method is based on the idea of multiply applying model reduction on certain LTI subsystems providing a reduced LTV system. The number of LTI subsystems and hence, the quality of the model reduction is controlled not directly but indirectly by an a posteriori error estimator for optimal control problems [4]. This estimator limits the error provided by solving an optimal control problem subject to a large-scale LTV system on basis of the corresponding reduced system. Therefore, the reduced system is indirectly guaranteed to be a suitable approximation for the large-scale system. The method will be illustrated by industrial applications arising in the simulation of laser welding with a moving laser source.

References

[1] Bunse-Gerstner A, Kubalinska D, Vossen G, Wilczek D. (2010). h_2-norm optimal model reduction for large-scale discrete dynamical MIMO systems. *Journal of Computational and Applied Mathematics* **233**(5):1202–1216.

[2] Gugercin S, Antoulas AC, Beattie CA (2008). H2 model reduction for large-scale linear dynamical systems. *SIAM Journal on Matrix Analysis and Applications* **30**(2):609–638.

[3] Vossen G, Volkwein S (2012). Model reduction techniques with a-posteriori error analysis for linear-quadratic optimal control problems. *Numerical Algebra, Control and Optimization* **3**:465–485.

[4] Tröltzsch F, Volkwein S. (2009) POD a-posteriori error estimates for linear-quadratic optimal control problems. *Computational Optimization and Applications* **44**:83–115.

[1] Niederrhein University of Applied Sciences, Chair for Applied Mathematics and Numerical Simulation, Reinarzstr. 49, D-47805 Krefeld, Germany

An Inexact Trust-region Algorithm for Nonlinear Programming Problems with Dense Constraint Jacobians

List of authors:

A. Walther [1]

L.T. Biegler [2]

There is a wide range of applications where the derivative matrices of the corresponding minimization problems are of rather small size but dense. Examples for such a setting are Periodic Adsorption Processes (PAPs). Here, the purity of the product or the energy consumption serve as target function. Additionally, the state of the system is described by general nonlinear equality constraints. As a consequence, when using well-established techniques the run-time needed for the optimization of such systems may be dominated significantly by the computation of the dense Jacobian and its factorization.

This talk presents an alternative approach, namely an inexact trust-region SQP algorithm. The proposed method does not require the exact evaluation of the constraint Jacobian or an iterative solution of a linear system with a system matrix that involves the constraint Jacobian. Instead, only an approximation of the constraint Jacobian is required. Furthermore, it is assumed that an exact representation of the nullspace of the constraint Jacobian at the current iterate can be evaluated in a fixed finite number of steps if necessary. Corresponding accuracy requirements for the presented first-order global convergence result can be verified easily during the optimization process to adjust the approximation quality of the constraint Jacobian and its nullspace representation. A global convergence proof as well as first numerical results will be presented.

[1] University of Paderborn

[2] Carnegie Mellon University

Iterative methods for solving variational inequalities in Hilbert spaces

List of authors:
Rafał Zalas [1] Andrzej Cegielski [2]

Variational inequality problem, denoted by VIP(F, C), is one of the fundamental problems in optimization theory. In a real Hilbert space it is defined as a problem of finding such $\bar{u} \in C$ which satisfies inequality $\langle F\bar{u}, z - \bar{u}\rangle \geq 0$, with respect to all $z \in C$, where $F\colon \mathcal{H} \to \mathcal{H}$ is given monotone operator. Very often the subset C has a special structure. This subset is often the intersection of simpler to handle closed convex subsets or a sublevel set of a convex function or, more generally, a set of fixed points of a quasi-nonexpansive operator. In this talk we will consider an abstract variational inequality in a real Hilbert space, which covers all of these three cases. For this purpose we will introduce a class of approximately shrinking (AS) operators (compare with [1, Definition 16], quasi-nonexpansive operator $U\colon \mathcal{H} \to \mathcal{H}$, with $\operatorname{Fix} U \neq \emptyset$, is AS if $\|Ux^k - x^k\| \to_k 0$ implies that $d(x^k, \operatorname{Fix} U) \to_k 0$, for any bounded sequence $(x^k) \subseteq \mathcal{H}$). Moreover we discuss their basic properties. Finally, we will present a few examples of iterative methods with application of AS operators, which can be used to solve VIP(F,C). Iterative schemes which are going to be presented are mostly based on the hybrid steepest descent method introduced by I. Yamada in [3] and extended by A. Cegielski and R. Zalas in [1, 2]. These iterative schemes are related to cyclic, sequential and also to string averaging procedures of construction of operators, which are more general.

References

[1] A. Cegielski, R. Zalas, Methods for variational inequality problem over the intersection of fixed point sets of quasi-nonexpansive operators, *Numer. Funct. Anal. Optimiz.* **34** (2013) 255-283.

[2] A. Cegielski, R. Zalas, Properties of a class of approximately shrinking operators and their applications, *Fixed Point Theory - An International Journal on Fixed Point Theory, Computation and Applications*, accepted.

[3] I. Yamada, The hybrid steepest descent method for the variational inequality problem over the intersection of fixed point sets of nonexpansive mappings, in *Inherently Parallel Algorithms in Feasibility and Optimization and Their Application,* D. Butnariu, Y. Censor and S. Reich (eds.), Elsevier, Amsterdam, 2001, pp. 473–504.

[1] Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Poland
[2] Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra, Poland

Author index